EQUILIBRIUM NATIONAL BORDER AND ITS STABILITY

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Abstract
This article analyses a model in which the national border is determined non-dictatorially by being based on citizens’ preferences. Each country faces a trade-off in terms of social welfare when considering whether to increase its size. As a country’s size increases, the government can collect more taxes and provide more public goods, which, all else equal, makes its citizens better off. On the other hand, a country that increases its size is assumed to also increase the heterogeneity of its citizens’ preferences leading to increased mismatch between preferences and the public goods provided by government. Notwithstanding the benefit of greater quantities of public goods afforded by living in a larger country, greater dissatisfaction with the public goods provided by government (i.e. preference mismatch) makes some segments of the citizenry worse off. Contrary to Alesina and Spolaore (1997), we show that a symmetric national border may be unstable. We also examine whether voluntary declaration of nationality guarantees the social optimum. Despite economies of scale from unification, the model implies that, from a social welfare perspective, there may be either too few, or too many, countries. Unification leading to fewer countries can be social welfare enhancing if people’s preferences in those states are quite similar; secessions leading to a greater number of smaller states can be social welfare enhancing if preferences are sufficiently heterogenous.

Keywords: national border, stability, public goods, secession, unification
JEL Classification: H7

1. Introduction
Although changes in national borders and disputes over secession and unification are an observed regularity, they are difficult to predict. Given the potentially extraordinary costs of resolving disputes over national borders, we investigate the extent to which border disputes can be explained as a predictable instability based solely on heterogeneous preferences among potential beneficiaries of the public goods provided by government. Of course real-world border disputes are drawn on a larger and more complex set of differences. Nevertheless, the value of the thought experiment that our model entails is to emphasize that simple preference heterogeneity (without any out-group animus, religious conflict, ideology, or competing claims over property rights) provides a sufficient condition for border instability. Our model follows the intuition of early political philosophers who suggested that the nation, or government, can be understood as coming into existence to

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fulfill a collective need. In a Hobbesian anarchic state, for example, it can be claimed that the independent choices of individuals would tend to be unsuccessful in satisfying collective needs because of the (now widely studied) problem of free riding and socially inefficient provision of public goods. According to this view, government emerges as an attempt to forge an agreement or “social contract” that shifts away from inefficient decentralized provision and achieves Pareto-efficient provision of public goods such as national defense. ¹

Heterogeneous preferences, however, present a fundamental challenge to the possibility that government can provide a collective net benefit to all members. This challenge is especially formidable when a single bundle of public goods is to be provided. If the public goods are local in the sense that different bundles are offered at different locations, then within-country geographic movement (i.e. voting with one’s feet) can discipline local governments, as Tiebout (1956) first noted, thereby achieving efficient provision of local public goods. ² Similar phenomena could, in theory, occur across nations (in addition to the efficiency gains from differentiated public goods across local governments). In this paper, we consider the problem of determining national borders when people are allowed to choose their nationality. Tiebout's insight turns out to be useful in addressing the existence of national borders and their stability.

An individual's nationality is originally given by parents’ nationalities or, in some cases, the location of his or her birth. But the institutions that regulate immigration provide ample scope for individuals to change their nationality ex post. ³ Given the large and increasing number of people who reside in one country while retaining citizenship in another, voluntary choice of nationality by individuals may, arguably, become an increasingly important determinant of nationality relative to compulsion and the exogenously given conditions at birth. ⁴

Nationality provides both benefits and costs. On the one hand, a citizen can exercise a bundle of rights to enjoy the benefits of public goods. On the other hand, each citizen must (in many countries) pay taxes, complete mandatory military service and bear other costs to fulfill citizen obligations that are particular in each state. Each individual will choose the nationality that brings him or her the greatest net benefit. Such consideration of benefits and costs by both citizens and potential citizens induces competition among nations. Although the competition among nations has so far occurred only for a relatively small proportion of the world's population due to the high cost of switching nationalities, it is likely that the future will see continuing pressure for this range of competition among nations to expand. For example, immigration among EU citizens has increased following unification of European institutions that reduced the costs of moving across a (former) border. Given recent referenda over nationality in Scotland, Spain and elsewhere in Europe, together

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¹ See, for example, Bush (1972) and Buchanan (1975) on the motive of improving efficiency in the provision of public goods and Mueller (2003, Part I) on the origin of the state.

² “Clubs” play an almost equivalent role as “voting with one’s feet” in achieving Pareto efficiency. Both mechanisms rely on the voluntary association of individuals for collective action rather than voluntary exchange among individuals (Buchanan, 1965).

³ Of course, the individual's choice of government can be rejected by one or more governments. Thus, strictly speaking, the nationality of an individual is the outcome of a social contract that is made voluntarily between individual and nation based on the view that the nationality is a legal relationship.

⁴ According to the U.N., as of 1993, there were over 100 million people who lived in a different country from a home country and the number has kept growing continually.
with ongoing debates over representation and jurisdiction by European nation states versus the European Union, we are likely to see more contests between countries (and potential countries) for voluntary selection into nationality.

Cross-border migrations occur mainly because immigrants do not like the policies of their home country and want to enjoy larger benefits from the public goods (including policies) provided by the host country. For example, most refugees from North Korea or illegal immigrants into U.S. seek to become citizens of a different country (at least in large part) because they want better policies and public goods. One interesting question concerning the stability of national borders is whether a handful of North Korean refugees or sudden changes in immigration flows can trigger a massive exodus. Under what condition can a few refugees or immigrants become an avalanche?

To address these issues formally, we consider the Hotelling’s (1929) linear city model in which individuals are located uniformly on the unit interval [0,1]. Assuming that each individual can decide his nationality by himself, we will examine how the national border between two nations is determined and whether the border is stable. Individuals located between the two countries face a trade-off when they decide whether to join a larger country. If an individual joins a larger country, then his or her net benefit from public goods will be larger. On the other hand, this individual incurs a larger transportation cost to consume the public goods in the larger country (i.e. receives lower preferential rent by joining the larger, more heterogenous country with larger mean distance between citizens’ preferences and the public goods provided there). We show that the symmetric equilibrium always exists, but it may be either stable or unstable. Counterintuitively (perhaps), heterogeneity of preferences promotes stability, and too much similarity leads to instability. As the people in our model become more homogeneous, the exit of the individual located on the border to the other country makes the net benefit from enjoying public goods provided by the other country grow larger (relative to the cost of transportation) and, therefore, more likely to trigger the bandwagon of neighbours. This mechanism implies that the equilibrium national border maybe unstable.

The model also enables us to compare global social welfare in the contrasting cases of a two-country versus one-country world. There is the theoretical possibility of an efficiency gain from unification (i.e. a one-country world) due to avoiding duplicate investments by two states in non-rival public goods that can be provided just once by a single state (instead of twice by two states). There is also the possibility of efficiency losses in the one-country world (i.e. efficiency gains from secession), because individuals bear higher transportation costs on average, interpreted as greater mean mismatch between citizens' preferences and the public goods provided by the state. As individual preferences become more homogeneous, the one-country world is more likely to be optimal, because the efficiency loss associated with the one-country world attenuates. In the opposite direction, however, when preferences are variegated and heterogenous, the possibility emerges of too few countries in the world, which may provide at least a partial explanation for the disintegration of several Eastern European countries such as Czechoslovakia and Yugoslavia that aggregated citizens whose preference heterogeneity exceeded the benefits from having a larger country size. This possibility is due in part to a coordination failure. Global welfare could be higher if the citizens in a country were able to coordinate themselves through a collective deviation to the other country. However, collective deviation does not necessarily lead to the first-best outcome; it may be profitable for a small group of homogeneous people to form a separate
small country rather than to belong to a giant country that consists of heterogeneous people. Therefore, even if it is socially optimal that there exists only one country, such a one-country world may not be realized, although the one-country world is more likely to emerge as the population becomes more homogenous.

China may provide a relevant example. Since China adopted economic liberalization in ways that encouraged greater investment, private enterprise, and accumulation of private wealth, the distance (i.e., heterogeneity) separating the preferences and economic systems of China and Taiwan has likely been reduced. Reductions of preference heterogeneity in China and Taiwan would (according to the model) generate wider support for unification.5

The case of North and South Korea provides another interesting example to consider in the light of our model. There remains a significant gap between North Korea and South Korea in nearly all respects including the economic and political systems. Our model suggests that if North Korea were to become more open and therefore less different from South Korea, then the border would become more unstable implying that a small shock to flows of individuals out of North Korea and into South Korea would more likely lead to an avalanche of refugees and greater likelihood of unification.

In our model, the capacity of countries to provide preferential rents imposes a similar limit on country size. Individuals lucky to be located relatively close to the midpoint of their country (representing the seat of government from which public goods are distributed) enjoy greater surplus, because they receive the same public goods at lower individual cost than those who are located in the periphery (even though all citizens in our model pay the same taxes). Individuals who receive the public goods at lower gross cost receive a rent that rationalizes stronger support for the government (i.e., lower transportation cost is interpreted as valuing the basket of public goods provided in that country more, or facing lower transaction costs to receive the same level of government service that the law is supposed to guarantee for all citizens). A country’s size therefore cannot expand too much, because its citizens’ preferential rents may fall too low.

Larger country size always weakens support for that country among some sub-population. The country’s decision variable (i.e., location of the midpoint or capital of the country) affects all individuals’ thresholds that they use to make cost-benefit comparisons when choosing which country’s public goods are more attractive. Changing the location of government generally induces some citizens to want to switch nationalities. If a country chooses to locate its capital at a more distant location from a particular individual (who becomes aggregated into a larger, more heterogeneous, segment of the global population), then government becomes less valuable to that individual.

There has been a growing literature on country size and the formation of national borders. Friedman (1977) addresses the issue of the determination of sizes and shapes of countries by assuming that governments are Leviathans whose objective is to maximize revenue net of collection costs. In his model, due to the diseconomies of scale in collection costs, the size of a country is naturally determined. Alesina and Spolaore (1997) explore

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5 Since the 1990s, many new expressions of political support in Taiwan for unification have appeared, covering a larger range of the traditional political spectrum – from Li Ao’s policy of “One Country, Two Systems” to perhaps more surprising expressions of support from middle-of-the-road political voices who favour upholding the status quo until mainland China democratizes and industrializes to the same level as Taiwan.
the determination of national borders in the Hotelling’s model as in this paper.\textsuperscript{6} They study the optimal number of nations and the stable number of nations obtaining the result that only nations of the same size can be stable. Their result, which contrasts sharply with ours, relies on their assumption that expenditures on public goods are fixed (\textit{i.e.} public goods expenditure does not depend on country size).\textsuperscript{7} If two countries differ in size, then a small decrease in the size of the smaller country increases its \textit{per capita} cost burden of financing the public goods by more than it gains from the increase in homogeneity that smaller country size provides (reducing mismatch between citizen preferences and the public goods provided by government to match the median voter’s preferences). These incentives lead to unstable tipping toward unification with the larger country. Such tipping does not necessarily occur in our model, however, so long as expenditures on public goods increase with country size. If the sizes of the two countries are identical, the situation is reversed. If a marginal individual (indifferent between being a citizen of either country) joins a (slightly) larger country, he enjoys larger benefits from public expenditures unlike in the case of fixed public expenditures. Our model shows that this crucial difference in the structure of public expenditures (increasing in country size in our model but fixed in Alesina and Spolaore, 1997) generates different incentives that can cause the symmetric border equilibrium to lose its stability leaving it vulnerable to unravelling.

Staal (2006) addressed a quite similar issue modifying Alesina and Spolaore’s (1997) assumption that public goods expenditures are fixed with respect to population size. The main difference between his paper and ours is that Staal’s \textit{per capita} tax is exogenously given (as a function of country size), whereas it is endogenously determined in our model. Also, many of Staal’s results rely on assumptions that our model relaxes. In particular, government spending on public goods is assumed to be increasing in country size in Staal’s model; once again, this feature is not assumed in our model but rather is derived endogenously.

One of the main features that distinguishes our analysis from the rest of the literature is the way in which we model economies of scale.\textsuperscript{8} In most of the literature including Alesina and Spolaore (1997, 2006), Goyal and Staal (2004), Staal (2010) (with Staal, 2006, remaining as an important exception), the primary benefit of larger country sizes is smaller \textit{per capita} tax rates. However, economies of scale in providing public goods may occur on the benefit side as well as on the cost side. In our model, the main advantage of a larger country size comes from higher gross benefits thanks to the greater provision of public goods. This difference comes from alternative assumptions. In our model, the benefit from public goods is not fixed, and instead increases proportionally with the population size.

\textsuperscript{6} The Hotelling’s model was also used by Casella and Feinstein (1990) and Wei (1992a, b) in a similar context.

\textsuperscript{7} Alesina and Spolaore (1997) argue that their result is not affected by the alternate assumption, which we pursue in this article, that expenditures on public goods depend on country size \textit{(i.e. $a + \beta s$ where $a, \beta > 0$ and $s$ is the country’s population size)}. However, if public goods are financed entirely by taxes, then it is not possible that $a > 0$. And under the restriction $a = 0$, Alesina and Spolaore’s result of stable borders between symmetrically sized countries is not generalized to our case of size-dependent financing, which is obvious from the intuition on tipping provided above.

\textsuperscript{8} Some economists doubt the assumption of economies of scale in the provision of public goods. In particular, Friedman (2005) criticized that statistics about the expenditure in the national defense does not support the assumption of economies of scale in terms of \textit{per capita} tax, although total expenditure is on average lower relative to GNP for larger countries. Also, Rose (2006) argued that he could find little evidence supporting the scale effect, especially in national defense, education and healthcare. However, their criticisms are only valid in \textit{per capita} terms.
This can be justified by imagining that a more useful public project that would not be possible with limited tax resources could be launched with more tax resources. Insofar as one views this alternative assumption as providing a more realistic model (i.e., based on the assumption that potential benefits from collective action are increasing with population size), then it is the case that per capita net benefit from public goods increases with the country size, while it is not obviously true nor important whether per capita tax increases with the country size or not itself.

The paper is organized as follows. In Section 2, we set up the model. Section 3 provides the analysis characterizing the stable national border. Section 4 contains discussions on the global welfare of the stable border and globalization. Concluding remarks follow in Section 5.

2. Basic Model

The world population is distributed uniformly over an interval [0,1]. The population mass is normalized to one and the locations of individuals are fixed. Here, an individual’s location should be interpreted as his preference, not as his physical location. There are two nations (or countries), A and B. All individuals incur the unit travelling cost c to get benefits from the public goods provided by the government. Thus, if the governments are located at \( z_A \) and \( z_B \) respectively where \( 0 \leq z_A < z_B \leq 1 \), the total travelling costs for an individual located at \( i \) are \( c|z_A - i| \) and \( c|z_B - i| \), respectively. Although governments provide an array of public goods, it may be convenient to consider education as a specific example. For example, we might interpret \( z = 1 \) as an extremely conservative education policy (e.g., removing state funding or somehow inducing vigorous competition) and interpret \( z = 0 \) as an extreme education policy on the opposite side of the political spectrum (e.g., with the goal of achieving universal welfare while discouraging school competition).

The players in this game include the continuum of agents and two governments. The game has two periods that unfold as follows. First, each individual declares which of the two nations, A and B, he will belong to. As a result, the border between the countries is determined. Then, the nations determine the location \( z_A, z_B \) and the tax level \( t_A, t_B \) of their respective government. This order of moves (i.e., the government makes its decisions after individuals make their respective decisions about nationality) reflects the reality that government policies change more frequently than individuals typically change their nationalities.

The natural solution concept that will be employed in this sequential game is the subgame perfect Nash equilibrium. By sticking to this equilibrium concept, we are implicitly

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9 A more secure national defense such as Terminal High Altitude Area Defense (THAAD) is an example.

10 We use the terms “nation” and “country” interchangeably throughout the paper.

11 In the 2014 election for the office of Seoul Superintendent of education, a liberal candidate Hi-Yeon Cho beat his conservative rival, Seung-Duk Koh, with a pledge to promote fairness by increasing the number of ordinary public high schools while reducing the number of elite schools.

12 Although we allow the possibility of double nationality, no one will choose it in this model, as far as the list of public goods in two countries overlaps significantly.

13 This sequence of moves can be also justified by casual observations that a regular stream of immigrations often changes policies of the host country. A flow of illegal immigrants into the U.S. is a typical example. Similarly, the number of refugees from North Korea has grown significantly since the 1990s leading to important changes in policy by the South Korean government including subsidized medical service, housing subsidy, private education, etc.
assuming that each individual makes a decision based on the belief that no other individual changes his nationality (i.e. taking the sizes of nations as given). Thus, each one will make a best response to the given nation size. Furthermore, subgame perfectness requires backward induction, which implies that individuals are forward looking in the sense that they choose nationality by taking into account this decision's effect on the location and the tax level of their government.

The role of the government is to provide a bundle of public goods and to collect taxes from its citizens. We assume that the public goods yield the same utility for all individuals, which is increasing in the aggregate tax amount, \( T \), that is, \( g = g(T) \) where \( g' > 0, g'' > 0 \) and \( g(0) = 0 \). Here, we implicitly assume that all the taxes collected are spent on the provision of public goods as the government expenditure. The more taxes are collected, the more public goods can be provided. Hence, it is quite natural that \( g'(T) > 0 \). Moreover, the government invests in public projects in the order of their value. Therefore, it is also natural that \( g'(T) \) is decreasing in \( T \).

The net valuation of individual \( i \) from belonging to country \( j \) is given by

\[
V_i(j) = y - c d_{ij} + g(T_j) - t_j
\]

where \( y \) is his income, \( d_{ij} \) is the distance from individual \( i \) to government \( j \) and \( t_j \) is the tax paid to government \( j \), whereas assuming that the income of each individual is the same. Also note that \( T_j = s_j t_j \) where \( s_j \) is the size of country \( j \).

Each country locates its government and chooses the (per capita) tax level to maximize the sum of its citizens’ net valuations given its country size.

3. Analysis

Following the requirement of the subgame perfect Nash equilibrium, we will use backward induction. Let \( s \) represent the size of country \( A \). Then it is clear that each nation will locate its government in the middle of its territory for any \( s \). Because the nation’s objective

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14 Note that one individual is negligible (i.e. has measure zero relative to the population) in this model with a continuum of individuals.

15 Since the nation size is a summary variable containing all the information of each individual decision, it is equivalent to the Nash equilibrium for each individual to make a best response to the nation size rather than the decisions of all others. This observation will be conveniently utilized in characterizing the stable national border.

16 This monotonic relationship also implies that if the aggregate tax is increased, then each citizen receives a greater benefit thanks to the richer set of public goods provided by government.

17 A seminar participant criticized this modelling approach, arguing that it is unnatural to assume that a benevolent government chooses the location and amount of the public goods in such a way that the aggregate utility of the individuals in its country is maximized, whereas individuals are assumed to be free to choose which country to join. However, as stated in Footnote 21, the government's decisions over location and the amount it spends on public goods can be interpreted as a result of majority voting. In our approach, government's decisions are essentially collective decisions, whereas the choice of nationality is an individual decision, the regulation of which cannot be easily justified.

18 If the order of the moves between individuals and governments is reversed, each government must then determine its location and tax rates without knowing its size and tax base; therefore, the subsequent size of country \( A \) may end up being either less or more than \( 2s_A \), depending on the relative expenditures on the public goods. If \( s \) is not equal to \( 2s_A \), then the government is tempted to change its
of maximizing the sum of its citizens’ utilities is equivalent to minimizing the sum of their disutilities from travelling, the problem of nation $A$ can be expressed as:

$$\min_{z} L_A(z) = \int_{0}^{s} (z - i)di + \int_{s}^{z} (i - z)di = -sz + \frac{s^2}{2}.\nonumber$$

Therefore, the optimal location is $z_A = s/2$. Similarly, country $B$’s optimal location is $z_B = s + (1 - s)/2 = (1 + s)/2$.\footnote{Alternatively, one can assume that the location of the government can be determined by the majority voting of the citizens. Then, $z_A = s/2$ and $z_B = (1 + s)/2$ can be interpreted as the voting outcome rather than the choice by the government. By the median voter theorem, the public good corresponding to the preference of the median voter (middle point) in each country is picked by majority voting.}

Let $G(t; s) \equiv g(st) - t$ represent the net valuation that each individual enjoys from the public goods. Then, the government chooses the optimal tax to solve the following problem:

$$\max_{t} G(t; s) = g(st) - t.\nonumber$$

The optimal tax $t'(s)$ must satisfy the first-order condition:

$$sg'(st) = 1. \quad (2)\nonumber$$

To develop an expression measuring the effect of a change in $s$ on $t'(s)$, we differentiate Equation 2 with respect to $s$ to obtain:

$$\left( g' + st'(s)g'' \right)ds + s^2g''dt = 0.\nonumber$$

The equation above implies that the effect is ambiguous, because $g' > 0$ and $g'' > 0$. If the country size is increased, it directly increases the marginal benefit from a tax increase of the country by $g'$, but indirectly decreases the marginal benefit due to an increase in the total government expenditures by $st'(s)g''$. If $|g''|$ is small (i.e. $g'$ does not change very much), then the former effect dominates the latter, leading consequently to greater taxes.\footnote{Many function forms of $g/(T)$ fall under this case. For example, consider $g(T) = aT^r$ where $r < 1$. Then, we can show that $t'(s) = (ar)^{1/(1-r)}s^{r/(1-r)}$, implying that $dt'(s)/ds > 0$.} However, the total tax revenue $T'(s) = st'$ is always increasing in $s$, because $g'(T^*)$ is assumed to be decreasing.\footnote{This result is consistent with the empirical finding by Alesina and Wacziarg (1998) that the amount of public spending is increasing in country size.} Let $G^*(s) \equiv G(t^*(s); s)$ be the optimized net valuation from public goods.

The net valuation of each individual in his country is the net valuation from consuming the public goods minus the traveling cost (locational disutility). Since the optimal location to each government is the midpoint of the segment encompassing its citizenry, the locational disutility is the lowest for the individual located in the middle of each country and the highest for the individual located at the border. Thus, we will say that the individual in the middle enjoys the highest “preferential rent”. The individual who enjoys this preferential rent is the one who has the (fixed) average preference of the population, because the government will optimally choose such a public good that corresponds to the average preference (or, equivalently, median preference, given the uniform distribution location to $s/2$. Except for the unrealistic case in which governments can commit to their locations no matter which nationalities individuals choose, our sequence of moves seems to fit better with intuition about who is primarily reacting to whom, although of course the model's fixed order of moves permits calculation of equilibrium in which both sides are best responding to each other.
of individuals) of the citizens. The farther an individual is located from the center, the lower is his or her preferential rent. In case of education policy for example, the citizen who has the moderate (i.e. median) view on education can enjoy the highest preferential rent from the education service provided by the government; the individual with median education preferences in his or her country will be maximally satisfied, because the education service actually provided must correspond exactly to what he or she desires. The preferential rent is sustainable due to the heterogeneous preferences of individuals. If all individuals are homogeneous, preferential rents would disappear, because the optimal location would be identical to the location of each individual. As the size of a country gets larger, the sum of individual preferential rents will be lower due to a higher degree of heterogeneity, while the benefit from public goods is increased. This is the main trade-off that a country faces.

To characterize the equilibrium border, consider the utility of the individual on the border. His utility when he belongs to country \( j \) is:

\[
V(j) = y - cs_j / 2 + G^*(s_j).
\]  

(3)

His choice between the two countries depends on the relative size of \( cs - 1/2 \) and \( G^*(s) - G^*(1 - s) \). The nationality of a larger country brings him both advantage and disadvantage. The advantage is attributed to the larger public expenditures; this advantage is measured by \( G^*(s) - G^*(1 - s) \). On the other hand, the disadvantage comes from greater disutility of travelling, which is measured by \( cs - 1/2 \).

Let \( \psi(s) \equiv G^*(s) - G^*(1 - s) \), \( \phi(s) \equiv cs - 1/2 \) and \( \Delta(s) = \psi(s) - \phi(s) \). If \( \Delta(s) > 0 \) (i.e. the size advantage of country \( A \) exceeds its locational disadvantage), then the individual on the border will strictly prefer country \( A \). This strict preference of the border individual implies that nearby individuals (i.e. sufficiently close to the border individual) will also prefer country \( A \) by continuity and, consequently, \( s \) cannot be the equilibrium size of country \( A \) whenever \( \Delta(s) > 0 \). If \( \Delta(s) > 0 \) (i.e. \( \psi(s) < \phi(s) \)), then the border individual will strictly prefer country \( A \) and \( s \) cannot be an equilibrium size of country \( A \) either. This argument leads to the following lemma.

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22 If the government expenditures on public goods are fixed, the benefit from a larger country size should be cost sharing. But, in this model, the amount of public expenditures is not fixed, so more citizens do not necessarily imply a lower tax per citizen. The benefit of a larger country size here is from enjoying more public goods. Criticisms against scale effects by Friedman (2005) and Rose (2006) are not applicable either, because what really matters to each citizen is the aggregate expenditures on public goods, not per capita expenditures, due to the non-excludability of public goods.

23 A similar insight can be found in the Tiebout model. Hillman (2009; p. 200) compares the case of a single jurisdiction with the case of two separate jurisdictions in the Tiebout model. In the case of a single jurisdiction, the whole population, which is divided into two groups (i.e. a high-benefit group and a low-benefit group), pays equal shares of the cost of the public goods. However, in the two jurisdiction case, the cost of the public goods is shared by smaller numbers of people implying greater per capita taxes in each jurisdiction (relative to the case of a single jurisdiction case); despite paying greater per capita tax, people in each jurisdiction benefit by enjoying the benefits of public goods that are more agreeable, or more closely match, their preferences. Therefore, there is a trade-off between cost sharing and preference satisfaction when it comes to public goods and the number of states or jurisdictions.

24 The scale effect clearly exists in the education sector if we interpret it as providing a greater variety of choices as in Footnote 16. For example, according to the U.S. Department of Education, the number of public elementary schools in the U.S. was 67,086 in 2010–2011, which is much larger than the 5,778 in Korea and 540 in Hong Kong.
Lemma 1 \( s \) is an equilibrium country size if and only if \( \Delta(s) = 0 \).

Proof. See the Appendix.

Because \( \Delta(s) \) is defined for the border individual implied by \( s \), Lemma 1 implies that it is sufficient to check the incentive of only a single (border) individual when characterizing an equilibrium border. This convenient shortcut follows from the continuity of the net benefits function representing individual preferences.\(^{25}\)

It is straightforward to see that \( s = 1/2 \) is an equilibrium border, although it may not be the unique one. Letting \( \bar{t} \equiv \text{argmax } g(t) - t \) and \( \hat{t} \equiv \text{argmax } g(t/2) - t \), we have:

Proposition 1 (i) The symmetric country size is an equilibrium one. (ii) It is not a unique equilibrium size if \( 2G(\bar{t}) < c < 4\hat{t} \).

Proof. See Appendix.

Now, let us turn to the issue of stability. We will call \( s \) a stable equilibrium border if for any small perturbation of the border \( s + \varepsilon \), no individual \( i \in (s, s + \varepsilon) \) has an incentive to deviate from \( s \); that is, sticking to \( s \) is the best response to any perturbed border \( s + \varepsilon \).\(^{26}\)

Under this definition, a stable border will be robust to a small perturbation. If the condition is not satisfied, then such perturbations would persist, and \( s \) would not be stable.

We begin by examining whether \( s = 1/2 \) is stable. To compare the slopes of \( \psi(s) \) and \( \phi(s) \), we first note that:

\[
\psi'(s) = \frac{\hat{t}^*(s) + \hat{t}^*(1-s)}{s(1-s)}
\]

which implies that \( \psi'(1/2) = 4\hat{t} \). Because \( \phi'(1/2) = c \), the condition for the equilibrium \( s = 1/2 \) to be stable is satisfied if \( c < 4\hat{t} \) and the equilibrium border is unstable if \( c < 4\hat{t} \).

The intuition is that if \( c < 4\hat{t} \), then the benefit from increased public goods increases faster than the larger country's locational disadvantage; thus, when the border is unstable, it means that in any perturbation where \( s > 1/2 \), then individuals near the border will best respond by moving to the larger country, implying that \( s \) becomes larger still. The possibilities of an unstable symmetric border or a stable asymmetric border in our model contrast with the result of Alesina and Spolaore's (1997) model in which only the symmetric border can be stable. The difference comes from their assumption that each nation incurs a fixed cost to organize the government and provide public goods. This fixed cost, \( k \), must be shared equally by the citizens of size \( s \), so that the per capita tax burden is \( k/s \). Note that any difference in per capita taxes between the two countries should be balanced with corresponding differences in travel cost at any equilibrium border. When \( s \) is perturbed marginally from the equilibrium state, the marginal individual in a world with Alesina and Spolarore's cost function for public goods will always find it in his interest to belong to a larger country if the country sizes differ (even if slightly), because the per capita share of tax would decrease

\(^{25}\) To elaborate on this, if \( \Delta(s) > (<) 0 \), individuals close enough to the border individual will have the same incentive as the border one, so there will be a group of individuals (i.e. a sense subset of the population with strictly positive measure) that all want to deviate from the proposed border, although the action of the single border individual itself is negligible.

\(^{26}\) The concept of the stable Nash equilibrium employed here is similar to the trembling-hand perfect equilibrium (Selten, 1975) or strategic stability (Kohlbarg and Mertens, 1986) in the sense that the decisions of players should be the best responses to slightly perturbed decisions of other players as well as to their equilibrium decisions. Alesina and Spolaore (2006) use a similar concept of stability.
geometrically while the travelling cost would increase linearly. Note that this insight is not robust to our alternate assumption that the cost of organizing the government depends on the country size, say $k(s)$, such that the ratio $k(s)/s$ does not decrease rapidly in $s$. Next we show that there must be a stable asymmetric border if $c > 2G(\bar{t})$.

**Proposition 2** (i) The symmetric country size is not stable if $c < 4\hat{t}$. (ii) There must be at least two asymmetric stable borders if $2G(\bar{t}) < c < 4\hat{t}$.

*Proof.* It is straightforward from the proof of Proposition 1.

For example, consider $g(T) = aT^n$. If $r = 1/2$, then the border is stable if $c > 4\hat{t} = a^2 / 2$ and unstable if $c < a^2 / 2$. This possibility of the existence of stable asymmetric borders is illustrated in Figure 1.

![Figure 1 | Stable Asymmetric Borders](image)

Source: authors' calculations based on the assumptions explicitly stated in the exposition of the model

Proposition 2, part (i), implies that an equilibrium border between two countries of symmetric size can nevertheless be unstable if their constituents are sufficiently homogeneous. Given sufficient preference homogeneity, even a small number of cross-border migrations can trigger a mass exodus.

4. Discussion

In this section, we undertake further analysis of the implications of the two propositions reported in the previous section with respect to global efficiency of two-country versus one-country regimes and discuss the extent to which consumer choice provides a reasonable analogy for analysing choice over nationalities.

**Welfare**

Global social welfare can be defined by the sum of all individuals’ net valuations in the world. In the abstraction of our model, “all individuals in the world” is represented by the
continuum along the unit interval, and global social welfare is simply the sum of net benefits over the all countries that partition the unit interval into subsegments. To demonstrate how global social welfare depends on the number of countries, we compare the cases of two and then one country, which illustrates the main trade-off in our model whenever the number of countries declines: there are efficiency gains from elimination of duplicate provisions of particular public goods by multiple countries traded off against efficiency losses from mismatch between preferences for public goods and the public goods actually provided (i.e., the one-world regime increases average preference mismatch among citizens of the larger politically unified country and the median preferences of its government).

Global welfare given \( s \) is represented by \( W(s) \), which can be defined as follows:

\[
W(s) = B(s) - L(s).
\]

where \( B(s) \) is the sum of net benefits of all citizens from public goods and \( L(s) \) is the sum of disutilities from travelling. Straightforward calculations show:

\[
B(s) = s \max_i \left[ g(st) - t \right] + (1-s) \max_i \left[ g((1-s)t) - t \right] = s G^*(s) + (1-s) G^*(1-s),
\]

\[
L(s) = c \left[ \int_0^{s} \left( \frac{s}{2} - i \right) di + \int_s^{1} \left( -\frac{s}{2} - i \right) di + \int_{s}^{1+s} \left( \frac{1+s}{2} - i \right) di + \int_{1+s}^{1} \left( i - \frac{1+s}{2} \right) di \right]
\]

\[
= \frac{2s^2 - 2s + 1}{4} c.
\]

Suppose that each government maximizes the sum of its citizens’ net valuations. Therefore, the government of country \( j \) implements its social optimum by choosing its tax and location optimally as a function of its size \( s_j \).

First, we will show that the global social welfare is larger in the symmetric equilibrium than in an asymmetric equilibrium.

**Proposition 3** In a two-country world, the global welfare is largest when \( s = 1/2 \), as long as there is a unique interior social optimum.

**Proof.** See the Appendix.

The intuition is that the total travelling disutility is minimized and the total net valuation from public goods is maximized when \( s = 1/2 \).

Could global welfare be increased if there were only one country in the world instead of two? Intuitively, the advantage of the one-country world over the two-country world is that it avoids expenditure on redundant public goods in two countries. The redundancy is captured in the assumption that \( g''(T) < 0 \). The main disadvantage of the one-country world is a high average travelling disutility (or, equivalently, low preference compatibility in the larger politically unified state). Since this disutility is proportionate to the unit travelling cost \( c \), there will exist an upper bound \( cT > 0 \) guaranteeing that whenever global preferences are sufficiently homogeneous such that \( c < c \), then global welfare in the one-country world is higher than in the two-country world.

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27 This social optimum is not global but local in the sense that it is the optimum for each country, but not the optimum for the world.
Proposition 4 Global welfare in the one-country world is greater than in the two-country world if \( c < 8\Delta B \equiv B - B(\frac{1}{2})(> 0) \), where \( B = \max_{t}(g(t) - t) \) and 
\[ B(s) = s \max_{t}(g(st) - t) + (1 - s) \max_{t}(g((1 - s)t) - t). \]

Proof. See Appendix.

The parameter \( c \) can be interpreted as measuring the costs of preference heterogeneity (i.e. an inverse measure of homogeneity in preferences) or how strong or particular the preferences of people across the world are. Proposition 4 implies that a one-country world is socially optimal only if people's preferences are sufficiently homogeneous.

Globalization

So far, we assumed that each individual makes a decision about nationality on the presumption that others do not change their nationalities (following the spirit of Nash equilibrium and related equilibrium concepts). Next, we allow for the possibility of a collective deviation\(^{28} \) and investigate conditions under which two countries become one country.\(^{29} \)

We first consider the incentive for a joint deviation away from two symmetric countries to one global country.\(^{30} \) Among the citizens who are least likely to want to deviate to the global country are those at \( x = 0,1 \), which are the locations that are most distant from the median of the global country implying that these agents would bear as large a cost of preference mismatch as anyone when shifting from the two-country to a one-country world. The gain from a collective deviation to one country is 
\[ G^*(1) - G^*(\frac{1}{2}) = \Delta B - B(\frac{1}{2}) \equiv \Delta B, \]
because 
\[ B(s) = sG^*(s) + (1 - s)G^*(1 - s). \]
Because disutility from preference mismatch (travel cost) increases from \( c/4 \) to \( c/2 \), the agents at \( x = 0,1 \) will join in the collective deviation if \( c < 4\Delta B \), and so will everyone else.\(^{31} \)

---

\(^{28} \) The idea of allowing for collective or joint deviations dates back to Aumann (1959). Despite its history and logical appeal (especially in settings with high degrees of social interdependency such as disputes in border regions), the concept of joint deviations has rarely been used, in part, because joint deviations leads to severe challenges of tractability when seeking to demonstrate the existence of equilibrium. This concept also clearly violates the spirit of the non-cooperative game theory. The joint deviation concept has been used, however, by Jackson and Wolinsky (1996), and others since, particularly in modelling network formation. There are typically too many equilibria in network formation problems, which motivates attempts to pare down the equilibrium set by allowing for additional types of deviations. Network formation is one example where people commonly make joint decisions and, therefore, joint deviations as a solution concept make intuitive sense. Group formation and nation formation are another context in which, we argue, the joint deviation concept has intuitive appeal.

\(^{29} \) Raising this issue of the unification of two or more smaller countries contrasts with economic research focussing more on separation into smaller states following, for example, the breakup of the former Soviet Union (e.g. Bolton and Roland, 1997; Bordignon and Brusco, 2001).

\(^{30} \) We are assuming that the consensus among all people in the two countries is required for the alliance to hold. This requirement is obviously too strong relative to the real-world alliances underlying political unifications. For example, the Treaty Establishing a Constitution for Europe requires majority (rather than unanimous) support of all constituent nations. See Jérôme and Vaillant (2005) for a detailed empirical analysis.

\(^{31} \) Consider the government's location decision in a single country versus corresponding decisions by governments in two countries by majority voting as interpreted in Footnote 21. If the alternatives are \( z = 1/4, 1/2 \) and \( 3/4 \), then \( z = 1/2 \) will beat the others by pairwise majority voting in the single
More generally, consider an asymmetric state \((s, 1-s)\) with \(s \leq 1/2\). For all \(s \leq 1/2\), the citizen who is least likely to deviate is \(x \approx 0\). Since his loss from preference mismatch (travelling cost) is \(\frac{1}{2}(1-s)c\) and his gain in net benefits from public goods is \(G^*(1) - G^*(s)\), he will deviate if \(\frac{1}{2}(1-s)c < G^*(1) - G^*(s)\), and so will everyone else.

Note that both the loss and the gain from deviation to the one-country world increases, the smaller the country is (i.e. the smaller \(s\) is). On the one hand, a citizen of a smaller country will be more reluctant to become a citizen of a global country, because he lives in a more homogeneous society implying lower cost of preference mismatch (travelling cost). On the other hand, this citizen's gain in the benefit from greater public goods by shifting to the larger country will also be greater. In particular, if \(s \approx 0\), then all the people will prefer the one-country world to the two-country world if \(c < 2(1) - 2\bar{B}\).

Furthermore, by using the monotonicity of \(\frac{1}{2}(1-s)c\) and \(G^*(1) - G^*(s)\) with respect to \(s\) (even though \(G^*(1) - G^*(s) - \frac{1}{2}(1-s)c\) may not be monotonic in \(s\)), we can say that a sufficient condition for everyone in a two-country world with arbitrary \(s\) to voluntarily give up their smaller states and collectively move to a one-country world is provided by the following upper bound on preference heterogeneity: \(c < 2\Delta B\).\(^{32}\)

It remains to compare the (globally) stable outcome\(^{33}\) with the global optimum. As shown before, the condition for the social efficiency of the one-country world is:

\[
c < 8\Delta B.
\]

Also, it was shown that the world will move from two symmetric countries to one country whenever \(c < 4\Delta B\). Therefore, if \(4\Delta B < c < 8\Delta B\), the result in our model is that two symmetric countries will not choose to be united into one global country even though it is socially efficient. That is, there may be too many countries. This result of underglobalization is due to the feature that one central government is too costly for the extreme citizen in

\(^{32}\) Some may wonder why a benevolent government does not prevent inefficient secessions and thereby increase aggregate utility of all individuals in the world whenever this condition is not satisfied. The default rule in this model is a two-country regime. The benevolent government is concerned about the welfare of its own citizens but not about the welfare of the citizens of the other country.

\(^{33}\) Of course, the one-country world may not be globally stable even if inequality (6) holds, because individuals may have a joint incentive to deviate from the one-country world by separating into more than two countries. We are implicitly assuming that there are at most two countries in the world.
terms of the travelling cost, although it is socially efficient in the sense that it reduces the average of travelling costs across citizens significantly.\textsuperscript{34} Note, however, that when the inequality in Proposition 4 fails to hold, then there can also be too few countries in the world and efficiency gains from secession into a larger number of smaller states capable of providing public goods that more closely match the place- and culture-specific preferences of each country’s citizens.

5. Conclusion and Caveats

In this paper, we characterized the equilibrium national border with new findings about its possible instability. Our model demonstrated the possibility of unstable border equilibria; that asymmetric equilibria exist; and that, depending on the relative importance placed on preference homogeneity within a state, there may be too few or too many countries according to a social efficiency criterion that aggregates net benefits across all individuals in the world. We provided a stability condition for the national border equilibrium in our model. To answer the question posed in the title of this article, the national border is unstable whenever individual preferences throughout the world become too similar or homogenous. Preference heterogeneity is the root cause of stable national borders in our model.

Although this paper provides several levels of equilibrium concepts and notions of stability, we acknowledge that those concepts are still rather limited. The major restrictiveness is that the joint deviation solution concepts we used did not take all possible deviations into account, because they allowed for at most two possible countries. A more satisfactory solution concept should be robust against any possible alternative pattern of nation formation. Examining more patterns of nation formation (allowing for an arbitrary number of countries) would provide a more thorough analysis with which to consider the full range of outcomes: the one-country world, the two-country world, and a world with a countable-infinity of countries (i.e. the possibility of Hobbes' anarchic outcome in which each individual is a state unite himself or herself). These challenging issues will be left for future research.

Also, for the purpose of characterizing the national border, we used the Hotelling’s model of horizontal differentiation and ignored the possibility of vertical differentiation by assuming that income levels are the same across all individuals. Policy choices by real-world governments (especially tax policy) are likely to have strong and differential effects according to income levels, thereby yielding potentially different levels of \textit{per capita} income in the two countries. In such a case, vertical differentiation of countries would be yet another factor to consider when analysing voluntary choice of nationality. This choice problem becomes challenging to analyse, however, because a smaller country (with higher \textit{per capita} income) may be able to provide greater quantities of public goods once we allow for income heterogeneity across countries. Although Bolton and Roland (1997) allowed some vertical differentiation in their model, they only focussed on the binary choice between two extremes—separation and unification—but did not consider the economic variables that determine the border in the first place. Incorporating vertical differentiation into our model of border determination will be another challenging issue that may be worth addressing in future work.

\textsuperscript{34} Goyal and Staal (2004) and Staal (2010) obtained similar results of excessive separation. However, their results are derived using the weaker solution concept of a majority voting rule. Our result is obtained using the unanimity rule.
Appendix

Proof of Lemma 1: This is clear, since the border individual is indifferent between $A$ and $B$ if and only if $\Delta(s) = 0$. Otherwise, for example, if $\Delta(s) > 0$, he would strictly prefer country $A$ over $B$, and so would all individuals $i \in (s - \varepsilon, s + \varepsilon)$ for some small $\varepsilon > 0$ by continuity. This would lead to a contradiction of the fact that all individuals $i < s$ prefer country $A$ while all $i > s$ prefer country $B$.

Proof of Proposition 1: (i) is clear. (ii) If $c < 4\hat{t}$, then we have $\phi'(1/2) = c < 4\hat{t} = \psi'(1/2)$. If $c > 2G(\hat{t})$, then $\Delta(1) = G(\hat{t}) - c/2 < 0$. Thus, by continuity of both $\psi(s)$ and $\phi(s)$, there must be at least one equilibrium border $s' \in (1/2, 1)$. (See Figure 1.) Also, by the symmetry of $\Delta(s)$ about $s = 1/2$, there exists at least one equilibrium border in $(0, 1/2)$.

Proof of Proposition 3: It is clear that $L(s)$ in Equation 5 is minimized, i.e. $\partial B(s)/\partial s = 0$ at $s = 1/2$. Now consider $B(s)$. By the envelope theorem, we have:

$$
\frac{\partial B(s)}{\partial s} = s \frac{\partial G^*(s)}{\partial s} + G^*(s) + (1-s) \frac{\partial G^*(1-s)}{\partial s} - G^*(1-s)
$$

$$
= s \left[ \hat{t}'(s) g'(st'(s)) \right] - (1-s) \left[ \hat{t}'(1-s) g'((1-s)\hat{t}'(1-s)) \right] + G^*(s) - G^*(1-s).
$$

Therefore, $\partial B(s)/\partial s = 0$ if $s = 1/2$. Hence, $W(s)$ is maximized at $s = 1/2$.

Proof of Proposition 4: Let the social welfare in the one-country world be $\bar{W} \equiv B - L$. We have:

$$
B(\frac{1}{2}) = \frac{1}{2} \left[ g\left(\frac{t}{2}\right) - t \right] + \frac{1}{2} \left[ g\left(\frac{t'}{2}\right) - t' \right] = g\left(\frac{t}{2}\right) - t.
$$

Thus, it follows that:

$$
\bar{B} = \max_{t} g(t) - t > \max_{t} g\left(\frac{t}{2}\right) - t = B(\frac{1}{2}),
$$

since $g(t) - t > g\left(\frac{t}{2}\right) - t$ for all $t$. Also, we have $L(s) = \frac{c}{8}$ and $\bar{L} = \frac{c}{4}$. Therefore, it follows that $\bar{W} > W(\frac{1}{2})$ if $c < \Delta B \equiv \bar{B} - B(\frac{1}{2})$.

References


