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SMART AGENTS AND SENTIMENT IN THE HETEROGENEOUS AGENT MODEL

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Abstract:
In this paper we extend the original heterogeneous agent model by introducing smart traders and changes in agents' sentiment. The idea of smart traders is based on the endeavor of market agents to estimate future price movements. By adding smart traders and changes in sentiment we try to improve the original heterogeneous agents model so that it provides a closer description of real markets. The main result of the simulations is that the probability distribution functions of the price deviations change significantly when smart traders are added to the model, and they also change significantly when changes in sentiment are introduced. We also use the Hurst exponent to measure the persistence of the price deviations and we find that the Hurst exponent is significantly increasing with the number of smart traders in the simulations. This means that the introduction of the smart traders concept into the model results in significantly higher persistence of the simulated price deviations. On the other hand, the introduction of changing sentiment in the proposed form does not change the persistence of the simulated prices significantly.

Keywords: heterogeneous agent model, market structure, smart traders, Hurst exponent

JEL Classification: C15, D84, G14

1. Introduction

An important feature of heterogeneous agents models (HAM) is their ability to explain stylized facts observed in financial time series, mainly fat tails and volatility clustering (see Lux and Marchesi, 2000, Farmer and Joshi, 2002). Typically, in the heterogeneous agents model, two types of agents are distinguished: fundamentalists and chartists. Fundamentalists base their expectations about future asset prices and their trading strategies on market fundamentals and economic factors, such as dividends, earnings, macroeconomic growth, and unemployment rates. Chartists or technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions. A literature on heterogeneous

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agent models has developed, *e.g*. Brock, Hommes (1998), Chiarella, He (2000), more recently LeBaron (2006), Hommes (2006).

In our previous work – Vosvrda, Vacha (2002, 2003) – we introduced a concept of memory and learning into the model of Brock and Hommes (1998). We also introduced other extensions, such as stochastic formation of beliefs and parameters including memory length. Application of the Worst Out Algorithm (WOA), which periodically replaces the trading strategies with the lowest performance was introduced in Vacha, Vosvrda (2005, 2007). In Vacha, Vosvrda (2005) we showed how the memory length distribution in the agents' performance measure affects the persistence of the simulated price time series.

In this paper we introduce a new concept – smart traders. The idea of smart traders is based on the endeavor of market agents to estimate future price movements. By adding smart traders we try to improve the original heterogeneous agents model so it can better approximate real markets. Smart traders are designed to forecast the future trend parameter of price deviations using an information set consisting of past deviations. For simplicity, they are modelled to assume that the price deviations, defined by the model, are an AR(1) process and they use the maximum likelihood estimation method for forecasting. Thus, in our model we use two groups of traders: smart traders and a group of stochastically generated trading strategies. Furthermore, we introduce changes in sentiment, which we define as a shift in beliefs about the future trend of a newly incoming investor strategy on the market. This allows us to model trend-followers and contrarians. In this paper we use only the form of jumps in sentiment. Our main expectation is that the introduction of smart traders and changes in sentiment will change the simulated market prices significantly.

The first part of the paper concerns a heterogeneous agent model which is an extension of the Brock and Hommes (1998) model. The second part briefly introduces the implementation of smart traders into the heterogeneous agent model framework. The last part of the paper investigates how the presence of smart agents and jumps in sentiment qualitatively changes the market structure.

2. Model

The model presents a form of evolutionary dynamics called the Adaptive Belief System in a simple present discounted value (PDV) pricing model. The first part of this model was elaborated by Brock and Hommes (1998), the second part is our extension of the original model. Simulated capital market is a system of interacting agents who immediately process new information. Agents adapt their predictions by choosing from a limited number of beliefs (predictors or trading strategies). Each belief is evaluated by a performance measure. Agents on the capital market use this performance measure to make a rational choice which depends on the heterogeneity in agent information.

Consider an asset pricing model with one risky asset and one risk free asset. Let p_t denote the price (ex dividend) *per* share of the risky asset at time t (random variables at time $t+1$ are denoted in bold) and let $\{\mathbf{y}_t\}$ be the stochastic dividend process of the risky asset. The supply of the risk free asset is perfectly elastic at the

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gross interest rate R, which is equal to $1+r$, where r is the interest rate. Then the dynamics of the wealth are defined as:

$$\mathbf{W}_{t+1} = RW_t + \left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) z_t, \tag{2.1.}$$

where z_t denotes the number of shares of the asset purchased at time t. Let us further consider E_t and V_t as the conditional expectation and conditional variance operators based on the set of publicly available information consisting of past prices and dividends, *i.e.*, on the information set $F_t = \{p_t, p_{t-1}, ...; y_t, y_{t-1}, ...\}$. Let $E_{h,t}$ and $V_{h,t}$ denote the beliefs (or forecast) of investor type h about the conditional expectation and conditional variance. Investors are supposed to be myopic mean-variance maximizers, so that the demand $z_{h,t}$ for the risky asset is obtained by solving the following criterion:

$$\max_{z_{h,t}} \left\{ E_{h,t}\left[\mathbf{W}_{t+1}\right] - \frac{a}{2} V_{h,t}\left[\mathbf{W}_{t+1}\right] \right\}, \tag{2.2.}$$

where the risk aversion coefficient, $a > 0$, is assumed to be the same for all traders. Thus the demand $z_{h,t}$ of type h for the risky asset has the following form:

$$E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right] - a\sigma^2 \mathbf{z}_{h,t} = 0, \tag{2.3.}$$

$$z_{h,t} = \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2}, \tag{2.4.}$$

assuming that the conditional variance of excess returns is constant for all investor types

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) = \sigma_h^2 = \sigma^2. \tag{2.5.}$$

Let z_t^s be the supply of outside risky shares. Let $n_{h,t}$ be the fraction of investors of type h at time t. The equilibrium of demand and supply is

$$\sum_{h=1}^{H} n_{h,t} \left\{ \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2} \right\} = z_t^s, \tag{2.6.}$$

where H is the number of different trader types. In the case of zero supply of outside shares, i.e., $z_t^s = 0$, the market equilibrium is

$$Rp_t = \sum_{h=1}^{H} n_{h,t} \left\{ E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right] \right\}. \tag{2.7.}$$

In a market where all agents have rational expectations, the asset price is determined by economic fundamentals. The price is given by the discounted sum of future dividends

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$$p_t^* = \sum_{i=1}^{\infty} \frac{E_t[\mathbf{y}_{t+i}]}{(1+r)^i}. \tag{2.8.}$$

The fundamental price p_t^* depends upon the stochastic dividend process y_t. In the special case where the dividend process $\{y_t\}$ is an independent, identically distributed (IID) process, with constant mean $E_t\{y_t\} = \bar{y}$, the fundamental price is given by

$$p^* = \sum_{i=1}^{\infty} \frac{\bar{y}}{(1+r)^i} = \frac{\bar{y}}{r}. \tag{2.9.}$$

2.1 Heterogeneous Beliefs

This part deals with traders' expectations about future prices. As in Brock and Hommes (1998), we assume beliefs about future dividends to be the same for all trader types and equal to the true conditional expectation, *i.e.*,

$$E_{h,t}[\mathbf{y}_{t+1}] = E_t[\mathbf{y}_{t+1}], \qquad h = 1,...,H, \tag{2.10.}$$

In the case where the dividend process $\{\mathbf{y}_t\}$ is an IID process, $E_t\{\mathbf{y}_{t+1}\} = \bar{y}$, then all traders are able to derive the fundamental price p_t^* that would dominate in a perfectly rational world. Abandoning the idea of rationality and moving to the real world we allow prices to deviate from their fundamental value p_t^*. For our purposes, it is convenient to work with a deviation x_t from the benchmark fundamental price p_t^*, *i.e.*,

$$x_t = p_t - p_t^*. \tag{2.11.}$$

In general, beliefs about the future price $E_{h,t}[\mathbf{p}_{t+1}]$ have the following form:

$$E_{h,t}[\mathbf{p}_{t+1}] = E_t\left[p_t^*\right] + f_h(x_{t-1},...,x_{t-L}), \qquad \text{for all } h,t, \tag{2.12.}$$

where $f_h(x_{t-1},...,x_{t-L})$ represents a model of the market. A type h trader believes that the market price will deviate from its fundamental value p_t^*. The heterogeneous agent market equilibrium, defined in equation (2.7.), can be reformulated in deviations from the benchmark fundamental as

$$Rx_t = \sum_{h=1}^{H} n_{h,t} E_{h,t}[\mathbf{x}_{t+1}] \equiv \sum_{h=1}^{H} n_{h,t} f_{h,t}. \tag{2.13.}$$

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2.2 Selection of Strategies

Beliefs are updated evolutionarily. The selection is controlled by endogenous market forces (Brock and Hommes, 1997). The fractions of trader types on the market $n_{h,t}$ are given by the multinomial logit probabilities of discrete choice

$$n_{h,t} = \exp\left(\beta U_{h,t-1}\right) / Z_t, \tag{2.14.}$$

$$Z_t = \sum_{j=1}^{H} \exp\left(\beta U_{h,t-1}\right), \tag{2.15.}$$

where $U_{h,t-1}$ is the fitness measure of strategy *h* evaluated at the beginning of period t. As the fitness measure of trading strategies we use the moving averages of realized profits, where m_h denotes the length of the moving average filter. In a real market this parameter can be interpreted as the memory length or evaluation horizon for trading strategy *h* at time *t*. The fitness measure $U_{h,t}$ is defined as

$$U_{h,t} = \frac{1}{m_h} \sum_{l=0}^{m_h-1} \left[\left(x_{t-l} - Rx_{t-1-l} \right) \frac{\left(f_{h,t-1-l} - Rx_{t-1-l} \right)}{a\sigma^2} \right]. \tag{2.16.}$$

3. Trading Strategies

Brock and Hommes (1998) investigated simple linear rules with one lag with fixed g_h:

$$f_{h,t} = g_h x_{t-1} + b_h, \tag{3.1.}$$

which served as a basic framework for the heterogeneous agent models. In this paper we enrich the model by introducing smart traders, who are able to forecast these linear rules and thus are able to forecast the future trend parameter $g_{h,t}$, which is variant in time. In our model we have two groups of trading strategies. The first group consists of smart trading strategies which use simple linear regression predictions of $f_{h,t}$, and the second group comprises trading strategies which are generated stochastically and selected with the Worst Out Algorithm (WOA) during the simulations. These two groups $f_{h,t}^1$ and $f_{h,t}^2$ will be defined below.

3.1 Smart Traders

The idea of smart traders is based on the endeavor of market agents to estimate future price movements. By adding smart traders we try to improve the original heterogeneous agents model so that it provides a closer description of real markets.

The simplest way to implement this type of market behavior into the Brock and Hommes model is to use simple linear forecasting techniques. The smart trading strategies thus use maximum likelihood estimation of an AR(1) process to estimate the trend parameter $g_{h,t}$ for the next period. Smart traders thus assume that the

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deviations x_t follow an AR(1) process, and they base their forecasts of x_{t+1} on the information set $F_t = \{x_t, x_{t-1}, ..., x_{t-k-1}\}$. Then the trading strategies of the smart traders are defined as follows:

$$f^1_{h,t+1} = \widehat{f}_{h,t} = \varphi_1 x_{t-1}, \tag{3.2.}$$

where φ_1 is the estimated trend $\widehat{g}_{h,t}$. In the simulations, we use various types of smart traders with different lengths k of the information set F_t.

3.2 Stochastic Beliefs

The trading strategies of the second group, $f^2_{h,t}$, are generated stochastically. The trend parameter g_h and the bias parameter b_h of trader type h are realizations from the normal distribution $N(0,\sigma^2)$. In this paper we use N(0,0.16) and N(0,0.9), respectively. The memory parameter m_h of the trading strategy $f^2_{h,t}$ is a realization from the uniform distribution, specifically U(1,100). The memory parameter can be interpreted as the evaluation horizon for the trading strategy h.

Further on, the WOA periodically replaces the trading strategies that have the lowest performance level of the strategies presented on the market by new ones. Without loss of generality, this algorithm is constructed to evaluate and rank the performance of all the strategies from the second group after every 40th iteration in descending order. The four strategies with the lowest performance are then replaced by the newly generated strategies. The use of the WOA in the simulations can significantly change the price time series parameters and modify the behavior of investors on the simulated market – see Vacha, Vosvrda (2005, 2007).

When $m_h = 1$ for all types h, we get the Brock and Hommes model. If $b_h = 0$ and $g_h > 0$, the investor is called a pure trend chaser. If $b_h = 0$ and $g_h < 0$, the investor is called a contrarian. Moreover, if $g_h = 0$ and $b_h > 0$ ($b_h < 0$), the investor is said to have an upward (downward) bias in his beliefs. In the special case of $g_h = b_h = 0$, the investor is fundamentalist, *i.e.*, the investor believes that the price always returns to its fundamental value.

3.3 Stochastic Beliefs with Change of Sentiment

Subsequently, we investigate the impact of a change in market sentiment on the simulated price. We define a change in sentiment as a shift in beliefs about the future trend $g_{h,t}$. In this paper we will focus on the direct jump pattern of agents' sentiment. This can be seen as jumps in sentiment from optimism to pessimism and vice versa. We model changes in sentiment by jumps in the trend parameter g_h of a newly incoming investor strategy on the market. More precisely, the trend parameter g_h jumps between realizations from the normal distributions N(0.4, 0.16) and N(–0.4, 0.16) every 4,000 iterations. The changes in sentiment can be observed in Figure 1 in the next section.

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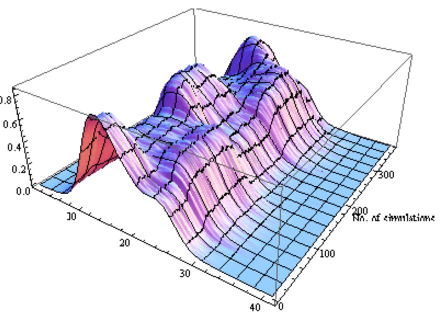
4. Simulation Results

This section describes the methodology of our simulations and summarizes the main results. The main purpose of the simulations is to examine the influence of the proposed smart traders concept and changes in sentiment on the simulated market prices. We compare the initial model without smart traders (0ST) with the model with five smart traders (5ST) and the model with five smart traders in the first group and changes in sentiment in the second group (5STS).

Altogether we consider 40 trading strategies for each simulation. For the model without smart traders, all the strategies are second group strategies $f_{h,t}^{2}$. For the simulations with five smart traders, there are five first group strategies $f_{h,t}^{1}$ and 35 second group strategies $f_{h,t}^{2}$. For the simulations with five smart traders and changes in sentiment, there are five first group strategies $f_{h,t}^{1}$ and 35 second group strategies $f_{h,t}^{2}$ where the trend parameter g_h jumps between realizations from the normal distributions *N*(0.4, 0.16) and *N*(–0.4, 0.16) every 4,000 iterations. Figure 1 shows the empirical probability density function of the trend parameter observed on the simulated market. It is the cross-section through the iterations, and the changes in sentiment can be clearly observed.

Figure 1
Empirical PDF of the Trend Parameters g_h through the Iterations



For trend parameter estimation by various types of smart traders we use different lengths $\{k_i\}_{i=1}^{5} = \{80, 60, 40, 20, 5\}$ for both models.

The other chosen parameters of the simulation are: β = 300 total number of iterations N = 15000, $a\sigma^2 = 1$, R = 1.1. Each of the six models has been simulated 36 times to achieve robust results. Table 1 shows the descriptive statistics of the simulated deviations x_t. Figure 2 shows the kernel estimation[1] of the probability density functions (PDFs).

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1 We use the Epanechnikov kernel, which is of the following form:
$K(u) = \frac{3}{4}(1-u^2)(|u| \le 1)$.

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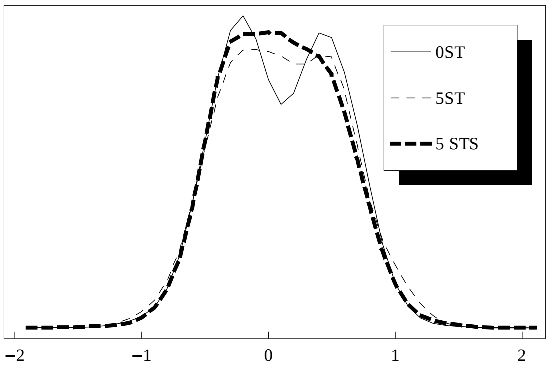
Table 1
Descriptive Statistics

Statistics	0 ST	5 ST	5 STS
Mean	0.00419734	0.0131363	-0.00717577
Median	0.00178642	0.00574712	-0.0233304
Variance	0.215544	0.228529	0.213265
St. Dev.	0.46168	0.474366	0.458831
Skewness	0.00228263	0.0421294	0.0957988
Kurtosis	2.54122	3.86883	4.32745
Min.	-2.97239	-6.01944	-7.57153
Max.	4.35099	9.53744	8.46828

Figure 2
Empirical PDF of x_t for Simulated Models without Smart Traders, with Five Smart Traders, and with Five Smart Traders and Changing Sentiment



We begin the results summary with the descriptive statistics of x_t. We can see that the means and variances of x_t do not change with an increasing number of smart traders. The models with smart traders produce leptokurtic distributions of x_t, while the model without smart traders produces a platykurtic distribution. While the values of skewness and kurtosis are the arithmetic means of all the simulations, we use the Kruskal-Wallis (1952) test to compare the distributions of simulated x_t. This test does not assume normal distribution of the compared sets of data. The null hypothesis of the test is an equal population of medians against the alternative of an unequal population of medians. The Kruskal-Wallis test rejects the null hypothesis of equal medians of the sets at the 10% significance level, thus the skewness and kurtosis of all three models are significantly different. We can conclude that adding smart traders as well as adding changes in sentiment to the original model significantly changes the simulated distributions of x_t.

For the measurement of persistence, we continue our analysis by estimating the Hurst (1951) exponent for all the simulated models. Before we discuss the results, we will briefly introduce the methodology. There are several methods to estimate the

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Hurst exponent. These are the R/S method (Hurst, 1951), periodogram method and Detrended Fluctuation Analysis (Peng *et al.*, 1994), to mention the basic three. In our paper we use only the R/S method that gives comparable results for financial time series, for detailed discussion of this topic see Lillo, Farmer (2004).

To estimate the Hurst exponent we use the R/S (Range-Scale) statistic. The basic idea is to compare the minimum and maximum values of sums of deviations from the sample mean, renormalized by the sample standard deviation. For the persistent or long-memory process the deviations are larger than for a random walk process. Detailed descriptions of the computational algorithm can be found in Hurst (1951), Peters (1994), and Los (2003).

A time series is a random walk when $H = 0.5$. This implies an independent innovation process, *i.e.*, there are no long memory effects in the observed time series. Conversely, when H differs from 0.5 the observations are not independent. A persistent or anti-persistent time series is characterized by long memory effects.

When $0 < H < 0.5$, we have an anti-persistent time series. It reverses itself more frequently than a random process. This behavior is very close to a mean-reverting process.

The interval $0.5 < H < 1$ describes a persistent process. The process is charterized by evident trends which are unpredictably interrupted by sharp discontinuities. The power of the trend-reinforcing behavior, or persistence, increases as H approaches unity. This long memory occurs regardless of time scale, *i.e.*, there is no characteristic time scale that is the key characteristic of a fractal time series and suggests a less efficient financial market in the sense of the Efficient Market Hypothesis (Peters, 1994 and Los, 2003).

Our expectation is that the introduction of smart traders and changes in sentiment into the simulated market will also increase the Hurst exponent significantly. We again use the Kruskal-Wallis test, which rejected the null hypothesis of equal medians of the estimated Hurst exponents at the 1% significance level for the compared models without smart traders and with smart traders. Thus we conclude that impalementing smart traders into the model significantly increases the Hurst exponent and thus increases the persistence of the simulated market. On the other hand, the Kruskal-Wallis test does not reject the null hypothesis of equal medians when comparing the 5ST and 5STS models. This means that the introduction of changing sentiment in the proposed form does not change the persistence of the simulated prices significantly.

5. Conclusion

In this paper we extended the original heterogeneous agent model by introducing the smart traders concept. The idea of smart traders is based on the endeavor of market agents to estimate future price movements. By adding smart traders we try to improve the original heterogeneous agents model so that it provides a closer description of real markets.

Smart traders are able to forecast the trend parameter of price deviations using an information set consisting of past deviations. They are modelled to assume that the

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deviations are an AR(1) process and they use the maximum likelihood estimation method for forecasting. Thus, in our model we use two groups of traders: smart traders and a group of stochastically generated trading strategies which are, moreover, selected by the Worst Out Algorithm. Furthermore, we investigate the impact of changes in sentiment of market participants on the simulated price. A change in sentiment is defined as a shift in beliefs about the future trend value.

The main result of the simulations is that the probability distribution functions of the price deviations change significantly with an increasing number of smart traders in the model, and they also change significantly when changes in sentiment are introduced. We also use the Hurst exponent to measure the persistence of the price deviations and we find that the Hurst exponent is significantly increasing with the number of smart traders in the simulations. This means that the introduction of the smart traders concept into the model results in significantly higher persistence of the simulated price deviations. On the other hand, the introduction of changing sentiment in the proposed form does not change the persistence of the simulated prices significantly. These are preliminary results, as more forms of changes in sentiment need to be tested to produce more general implications for real markets.

As this paper introduces a new approach for modelling heterogeneous agents, it also opens up considerable scope for further research. The most interesting aspect would be to show the impact of different changes in sentiment on the market price.

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