HETEROGENEOUS AGENT MODEL WITH MEMORY AND ASSET PRICE BEHAVIOUR

Miloslav VOŠVRDA, Lukáš VÁCHA*

Abstract:
The efficient markets hypothesis provides a theoretical basis on which technical trading rules (TTRs) are rejected as a viable trading strategy. TTRs, providing a signal to the user when to buy or sell asset based on such price patterns, should not be useful for generating excess returns. Technical traders tend to put little faith in strict efficient markets hypothesis. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as technical traders. Switching between the technical trader's and fundamentalist's strategy is a basis of the cycle behaviour. This event is analysed by the Brock and Hommes (BH) model. Moreover, the memory case is added to this model because BH model was the memory-less model. This branch consists of a behaviour analysis among fundamentalists and technical traders. Here is a basis for endogenous source of the real business cycle.

Keywords: efficient markets hypothesis, technical trading rules, fundamentalists, technical traders, chartists and contrarians, heterogeneous agent model with memory, asset price behaviour

JEL Classification: C61, G14, D84

1. Introduction

Assumptions about rational behaviour of agents, homogeneous models, and efficient market hypothesis were paradigms of economic and finance theory for the last years. After empirical data analysis on financial markets, economic, and finance progress, these paradigms are gotten over. There are phenomena observed in real data collected from financial markets that cannot be explained by the recent economic and finance theories. One paradigm of recent economic and finance theory asserts that sources of risk and economic fluctuations are exogenous. Therefore the economic system would converge to a steady-state path, which is determined by fundamentals and there are no opportunities for speculative profits in the

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*) Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 4, CZ – 180 00 Prague 8 (e-mail: vosvrda@utia.cas.cz; lvacha@cbox.cz).
**) Support from the Grant Agency of the Czech Republic under Grants 402/00/0439 and 402/01/0034 is gratefully acknowledged.

absence of external shocks prices. It means that the other factors play important role in a construction of real market forces as heterogeneous expectations. Since agents have no sufficient knowledge of the structure of the economy to form correct mathematical expectations, it is impossible for any formal theory to postulate unique expectations that would be held by all agents (see Gaunersdorfer, 2000). Prices are partly determined by fundamentals and partly by the observed fluctuations endogenously caused by non-linear market forces. This implies that TTR's need not be systematically bad and may help in predicting future price changes. Developments in the theory of non-linear dynamic systems have contributed to new approaches in economics and finance theory (see Brock, 2001). Introducing non-linearity in the models may improve research of a mechanism generating the observed movements in the real financial data. Financial markets are considered as systems of the interacting agents processing new information immediately. Heterogeneity in the agent expectations can lead to market instability and complicated market dynamics.

Our approach assumes that agents are intelligent having no full knowledge about the underlying model in sense of the rational expectation theory and having no computational equipment can interpret the same information by different way. Therefore prices are driven by endogenously market forces. The Adaptive Belief Approach by BH (see Brock, Hommes, 1997) is employed in this paper. Agents adapt their predictions by choosing among a finite number of predictors. Each predictor has a **performance measure**. Based on this performance measure, agents make a rational choice in a set of the predictors. BH showed that the adaptive rational equilibrium dynamics incorporates a general mechanism that may generate both a local instability of the equilibrium steady state and complicated global equilibrium dynamics.

We focus on a version of the model with two types of traders, i.e., fundamentalists, and technical traders. Technical traders tend to put little faith in strict efficient markets. Fundamentalists rely on their model employing fundamental information basis to forecasting of the next price period. The traders determine whether current conditions call for the acquisition of fundamental information in forward looking manners, rather than relying on post performance. This approach relies on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as technical traders. A changing of the technical traders's profitability and fundamentalist's positions is a basis of the cycle behaviour. A more detailed analysis is introduced in the BH model (see Brock, Hommes, 1997). This model is analysed under assumption that agents are without a memory. We analyse this model under assumption that agents have been using a memory. Moreover, different forms of memory processing are considered.

This paper is organized as follows. In section 2 we briefly introduce the BH model without memory. This branch consists of a behaviour analysis among fundamentalists, and technical traders. In section 3 the BH model is analysed with dynamics of fractions. In section 4 trading strategies as fundamentalists', and technical traders' are defined. The BH model is studied in this section with memory added in a performance measure. Section 5 is devoted to a numerical analysis of the model under different memory lengths for three significant different parameter's cases. Results are summarised in section 6.

2. Model

An analysed model presents a form of evolutionary dynamics, which is called **adaptive belief system**, in a simple present discounted value (PDV), for the pricing model. BH presented this model without memory and one period only (see Brock, Hommes, 1998).[1)]

Let us consider an asset-pricing model with one risky asset and one risk-free asset. Let p_t be the share price (ex dividend) of the risky asset at time t, and let $\{y_t\}$ be an independent identically distributed (i.i.d.) stochastic dividend process of the risky asset. The risk free asset is perfectly elastically supplied at gross return $R > 1$. The dynamics of wealth can be written as

$$W_{t+1} = R \cdot W_t + (p_{t+1} + y_{t+1} - R \cdot p_t) \,.\, z_t \tag{1}$$

where z_t denotes the number of shares of the asset purchased at time t. Let E_t and V_t denote the conditional expectation and conditional variance operators, based on the public available information set consisting of past prices and dividends, i.e., on the information set $F_t = \{p_t, p_{t-1}, \ldots; y_t, y_{t-1}, \ldots\}$. Let $E_{h,t}$, $V_{h,t}$ denote **beliefs** of an investor of the type h about the conditional expectation E_t and conditional variance V_t. A conditional variance of wealth is

$$V_{h,t}\,[W_{t+1}] = z_t^2 \cdot V_{h,t}\,[p_{t+1} + y_{t+1} - R \cdot p_t] \tag{2}$$

We assume that beliefs about the conditional variance of excess returns are constants for all investor types h, i.e.,

$$V_{h,t}\,[p_{t+1} + y_{t+1} - R \cdot p_t] \equiv \sigma_h^2 = \sigma^2 \tag{3}$$

Assume each investor type is a **myopic mean-variance maximizer.** So for type h, the demand for shares z_{ht} is solved as follows

$$\max_z \{E_{h,t}[W_{t+1}] - (a/2) \cdot V_{h,t}[W_{t+1}]\}, \tag{4}$$

i.e.,

$$E_{h,t}\,[p_{t+1} + y_{t+1} - R \cdot p_t] - a \cdot \sigma^2 \cdot z_{s,t} = 0, \tag{5}$$

$$z_{h,t} = E_{h,t}\,[p_{t+1} + y_{t+1} - R \cdot p_t] \,/\, (a \cdot \sigma^2) \tag{6}$$

A risk aversion, a, is here assumed to be the same for all traders. Let $z_{s,t}$ be a supply of shares per investor and $n_{h,t}$ the fractions of investors of type h at date t. The equilibrium among demand and supply is expressed in the following form

$$\sum_h n_{h,t} \{E_{h,t}[p_{t+1} + y_{t+1} - R \cdot p_t] / a \cdot \sigma^2\} = z_{s,t} \tag{7}$$

If there is only one type h, the market equilibrium yields the pricing equation

$$R \cdot p_t = E_{h,t}\,[p_{t+1} + y_{t+1}] - a \cdot \sigma^2 \cdot z_{s,t} \tag{8}$$

For the special case of zero supply, i.e., $z_{st} = 0$, for all t, a benchmark notion of the rational expectation fundamental price solution p_t^* is obtained. Then the expression (8) can be written in the following form

$$R \cdot p_t^* = E_t\,[p_{t+1}^* + y_{t+1}] \tag{9}$$

If the dividend process $\{y_t\}$ is an i.i.d., the expectation $E_t\{y_{t+1}\} = \bar{y}$, and a standard notion of **fundamental** is obtained. Let us put $p_t^* = \bar{p}$, where $\bar{p}$ is solution of

$$R \cdot \bar{p} = \bar{p} + \bar{y} \tag{10}$$

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1) The model was inspired by the Lucas's model (see Lucas, 1978).

The equation (9) has infinitely many solutions but only the constant solution $\overline{p} = \overline{y} \;/\; (R - 1)$ of the equation (10) satisfies no the bubbles condition, i.e., $\lim_{t \to \infty} E(p_t) / R^t = 0$. For our purpose, it is better to work with the deviation x_t from the benchmark fundamental price p_t^*, i.e.,

$$x_t = p_t - p_t^* \tag{11}$$

Heterogeneous beliefs will be now introduced and we shall study their influences on equilibrium of the dynamical systems. In this case of zero supply of outside shares, we get from the equation (7)

$$R \cdot p_t = \sum_h n_{h,t} \cdot E_{h,t}[p_{t+1} + y_{t+1}] \tag{12}$$

The class of beliefs for every trader type h must be specified. Therefore the following assumption is introduced. All beliefs are of the form

$$E_{h,t}\,[p_{t+1} + y_{t+1}] = E_t\,[p^*_{t+1} + y_{t+1}] + f_h\,(x_{t-1},\, \ldots,,\, x_{t-L}) \tag{13}$$

where p^*_{t+1} denotes the fundamental price, $E_{h,t}\,[p^*_{t+1} + y_{t+1}]$ is the conditional expectation of the fundamental on the information set F_t, $x_t = p_t - p_t^*$ is the deviation from the fundamental price, and f_h is some deterministic function which can differ across trader types *h,* i.e., **we restrict beliefs to deterministic functions of past deviations from the fundamental.** As a special case, the assumption includes the case of an i.i.d. dividend process with $E_t\,[y_{t+1}] = \overline{y}$ and the corresponding constant fundamental $p^*_t = \overline{p} = \overline{y} \;/\; (R - 1)$. We can rewrite the equation (12) in the deviations form

$$R \cdot x_t = \sum_h n_{h,t} \cdot E_{h,t}(p_{t+1} + y_{t+1}) - E_t(p^*_{t+1} + y_{t+1}) \tag{14}$$

using the equations (9), and (11) and the following form

$$R \cdot p_t = R \cdot x_t + R \cdot p_t^* \tag{15}$$

Now we use equation (13), and the fact that $\sum_h n_{h,t} = 1$ for all *t,* and we obtain

$$R \cdot x_t = \sum_h n_{h,t} \cdot \{E_t[p^*_{t+1} + y_{t+1}] + f_h(x_{t-1}, \ldots, x_{t-L})\} - E_t[p^*_{t+1} + y_{t+1}], \tag{16}$$

$$R \cdot x_t = \sum_h n_{h,t} \cdot f_h(x_{t-1}, \ldots, x_{t-L}) \equiv \sum_h n_{h,t} \cdot f_{h,t} \tag{17}$$

Denote the excess returns by expression $R_{t+1} = p_{t+1} + y_{t+1} - R \cdot p_t$. Let $\rho_{h,t} = E_{h,t}\,[R_{t+1}]$ be the conditional expectation of R_{t+1}. Let us consider the goal function

$$\max_z \{E_{h,t}[R_{t+1}] \cdot z - (a/2) \cdot z^2 \cdot V_{h,t}[R_{t+1}]\} \tag{18}$$

By using operators for expectations and variances we get

$$\max_z \{\rho_{h,t} \cdot z - (a/2) \cdot z^2 \cdot \sigma^2\}$$

The expression (18) is equivalent to the expression (4) up to a constant, so the optimum choice of shares of the risky asset is the same. Let us denote the optimum solution of the equation (18) by $z\,(\rho_{h,t})$.

3. Dynamics of Fractions

Let us concentrate on the adoption of beliefs, i.e., on dynamics of the fractions $n_{h,t}$ of different trader types. Next, let us change slightly the timing of updating beliefs

$$R \cdot x_t = \sum_h n_{h,t-1} \cdot f_h(x_{t-1},\dots,x_{t-L}) \equiv \sum_h n_{h,t-1} \cdot f_{h,t} \tag{19}$$

where $n_{h,t-1}$ denotes the fraction of trader type h at the beginning of period t, before than the equilibrium price x_t has been observed. Now **the realized excess return** over period from t till $t+1$ is computed,

$$R_{t+1} = p_{t+1} + y_{t+1} - R \cdot p_t, \tag{20}$$

$$R_{t+1} = x_{t+1} + p^*_{t+1} + y_{t+1} - R \cdot x_t - R \cdot p^*_t, \tag{21}$$

$$R_{t+1} = x_{t+1} - R \cdot x_t + p^*_{t+1} + y_{t+1} - E_t\,[p^*_{t+1} + y_{t+1}] + E_t\,[p^*_{t+1} + x_{t+1}] - R \cdot p^*_t \tag{22}$$

From the equation (9) we get

$$Et\,[p^*_{t+1} + x_{t+1}] - R\,.\,p^*_t = 0, \text{ and } \delta_{t+1} = p^*_{t+1} + y_{t+1} - Et\,[p^*_{t+1} + x_{t+1}]$$

which is a martingale difference sequence with respect to F_t i.e., $E_t\,[\delta_{t+1} \mid F_t] = 0$ for all t. So the expression (22) can be written as follows

$$R_{t+1} = x_{t+1} - R\,.\,x_t + \delta_{t+1} \tag{23}$$

The decomposition of the equation (23) as separating the "explanation" of realized excess returns R_{t+1} into the contribution $x_{t+1} - R\,.\,x_t$ of the theory is investigated here and the conventional efficient markets theory term δ_{t+1} is shown.

Let the performance measure $\pi(R_{t+1}, \rho_{h,t})$ be defined by

$$\pi_{h,t} = \pi(R_{t+1}, \rho_{h,t}) = R_{t+1} \cdot z(\rho_{h,t}) = (x_{t+1} - R \cdot x_t + \delta_{t+1}) \cdot z(\rho_{h,t}) \tag{24}$$

so the performance is given by the realized profits for the trader h. In the following paragraphs, numerical simulations with a stochastic dividend process $y_t = \bar{y} + \varepsilon_t$, where ε_t is i.i.d.,[2] with a uniform distribution on an interval $\langle -\varepsilon, +\varepsilon \rangle$ will be used.

Now write a type h beliefs $\rho_{h,t} = E_{h,t}\,[R_{t+1}] = f_{h,t} - R\,.\,x_t$ in the deviations form. Let the updated fractions $n_{h,t}$ be given by the discrete choice probability

$$n_{h,t} = \exp(\beta \cdot \pi_{h,t-1}) \,/\, Z_t, \text{ where } Z_t = \sum_h \exp(\beta \cdot \pi_{h,t-1}) \tag{25}$$

The parameter β is the intensity of choice measuring how fast agents switch between different prediction strategies. The parameter β is a measure of trader's rationality. The variable Z_t is just a normalization so that fractions $n_{h,t}$ sum up to 1. If the intensity of choice is infinite ($\beta = +\infty$), the entire mass of traders uses the strategy that has the highest performance. If the intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies.

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2) In this case we have $\delta_{t+1} = \varepsilon_{t+1}$.

The timing of predictor selection is important. The fractions $n_{h,t}$ depend upon the performance measure π and return R at the time $t-1$ in order to ensure that depends only upon observable deviations x_t at time t. The timing ensures that past realized profits are observable quantities that can be used in predictor selection.

4. Memory in the Performance Measure

For the case with memory in the performance measure, the performance measure is not given by the most recent past (last period), but by a summation of more values of the performance measure in the past with different weights for these values. The weights sum up to one:

$$n_{h,t} = \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p}\right) / Z_t,\ Z_t = \sum_h \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p}\right) \tag{26}$$

where m denotes the memory length, η is the vector of memory weights.

All beliefs will be of a simple form

$$f_{h,t} = g_h \cdot x_{t-1} + b_h, \tag{27}$$

where g_h denotes a trend and b_h a bias of trader type h.

If $b_h = 0$, the agent h is called a **pure trend chaser** if $g_h > 0$ (strong trend chaser if $g_h > R$) and a **contrarian** if $g_h < 0$ (strong contrarian if $g_h < -R$).

If $g_h = 0$, type h trader is said to be **purely biased**. He is upward (downward) biased if $b_h > 0$ ($b_h < 0$).

In the special case $g_h = b_h = 0$, type h trader is called **fundamentalist**, i.e., the trader believes that prices return to their fundamental value. Fundamentalists do have all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

Now we derive the performance measure for the simple belief type (2). Rewriting the equation (6) in deviations form yields the demand for shares by type h (by the assumption (13))

$$z_{h,t-1} = \frac{E_{h,t-1}[p_t + y_t - R \cdot x_{t-1}]}{a \cdot \sigma^2} = \frac{f_{h,t-1} - R \cdot x_{t-1}}{a \cdot \sigma^2} \tag{28}$$

Now the performance measure (24) can be rewritten hence the realized profit is

$$\pi_{h,t-1} = R_t \cdot z_{h,t-1} = (x_t - R \cdot x_{t-1} + \delta_{t+1})(g_h \cdot x_{t-2} + b_h - R \cdot x_{t-1}) / (a \cdot \sigma^2) \tag{29}$$

The most common trader type in our numerical analysis is fundamentalist with parameters $g_h = b_h = 0$. Hence for fundamentalists we can write

$$\pi_{F,t-1} = (x_t - R \cdot x_{t-1} + \delta_{t+1})(-R \cdot x_{t-1}) / (a \cdot \sigma^2) \tag{30}$$

where an index F denotes the fundamentalist investor type.

5. Numerical Analysis of the Model under Different Memory Lengths

This section demonstrates numerically an importance of the memory for behaviour of this model. We show that there are significant differences in profitability of trader's strategies as memory length is changed. In the second case and the third case we also use different memory lengths for various trading strategies, which also influence the traders profitability, i.e., the trader's participation on the market.

Here, a numerical analysis is focused only on the model with four types of trader's strategies, each with different beliefs. We examine three different parameter cases, where investor's types are fundamentalists that interact with other technical trader's types such as trend chasers, contrarians, or with both of them.

For all three cases in this section we add noise to a dividend process. A noise has a uniform distribution on the interval $\langle -0.005, +0.005 \rangle$. The equation (31) is used for a memory-less system, i.e., decision-making procedure is formulated using one period only, and the equation (32) for the system with a memory, where m denotes the memory length. The sum of memory weights $\eta_{j,p}$'s must add up to one.

Memory-less system generates the following price formulation

$$x_{t+1} = \frac{1}{R} \sum_{h=1}^{4} \left(\frac{\exp(\beta \cdot \pi_{h,t-1})}{\sum_{h=1}^{4} \exp(\beta \cdot \pi_{h,t-1})} (g_h x_t + b_h) \right) \tag{31}$$

System with memory, where m denotes memory length and η memory weights, generates the following price formulation

$$x_{t+1} = \frac{1}{R} \sum_{h=1}^{4} \left(\frac{\exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p} \right)}{\sum_{h=1}^{4} \exp\left(\beta \cdot \sum_{p=1}^{m} \eta_{h,p} \cdot \pi_{h,t-p} \right)} (g_h x_t + b_h) \right) \tag{32}$$

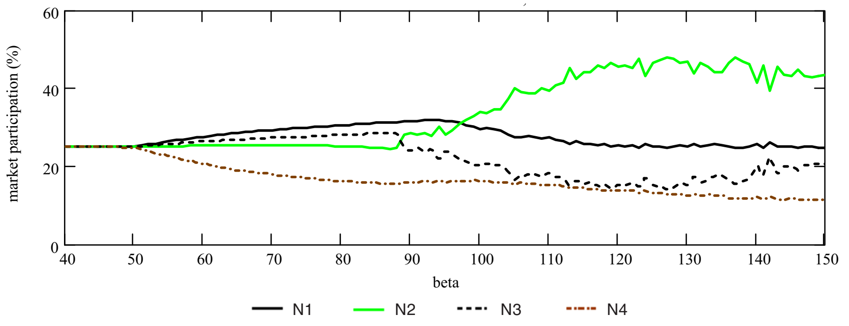
Case 1: Fundamentalists, Trend Chasers

This part of a numerical analysis of the system with parameters in Table 1 is without memory in the performance measure, i.e., agents make decisions according to the last period of the performance measure. For values of beta larger than 90 there arise chaotic price fluctuations and the trading strategy of trend chasers N2 (see Table 1) becomes dominant on the market (see Figure 1). We do not want to explore dynamic features in the sense of chaotic behaviour but mainly the presence of traders on the market.

Table 1
Parameters of the System for Case 1

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = 1.1$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = 0.9$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = 1$	$b_4 = 0$	Trend chasers

Figure 1
Participation of Trading Strategies on the Market with the Constant Memory Length (m = 1 for N1, N2, N3, N4, and different values of the parameter β)



An effect of different memory lengths (for all strategies) is displayed in Figure 2. There is a dramatic change at m = 2, where fundamentalists becomes dominant strategy when m = 18 where no price fluctuations occur and strategies are equally represented on the market. This example shows the stabilizing effect of memory for the system (see Barucci, 2000).

Figure 2
Participation of Trading Strategies on the Market with the Constant Parameter β = 150 and with Different Values of the Memory Length

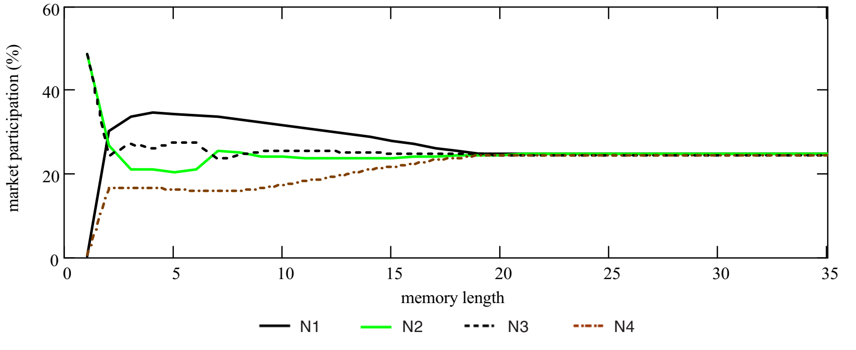
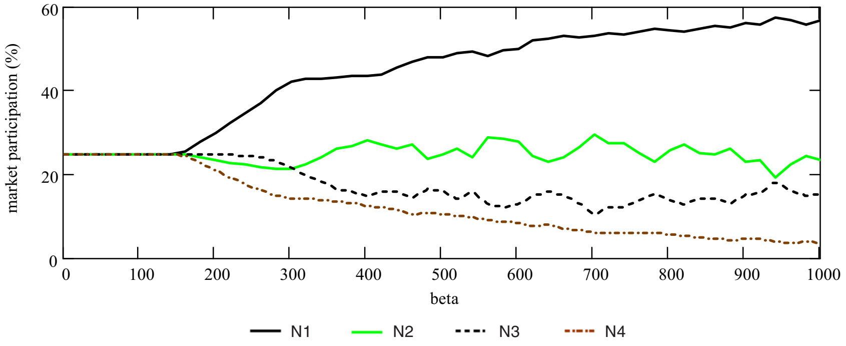


Figure 3 displays simulations with equal memory length for all trading strategies (m = 20), but with different values of the intensity choice parameter, β. The analysis shows remarkable result – with higher memory, the profitability of fundamentalists N1 on the market is rising.

Figure 3
Participation of Trading Strategies on the Market with the Constant Memory Length (m = 20 for N1, N2, N3, N4, and different values of the parameter β)



Case 2: Fundamentalists, Contrarians

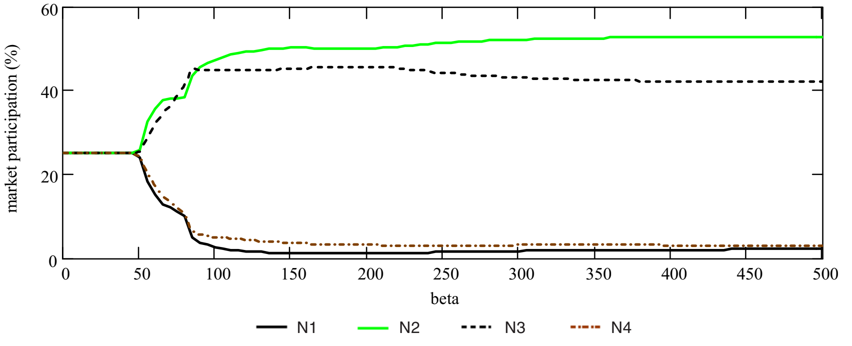
Next, we consider the case with four different belief types with parameters in Table 2. This case is a little bit specific one because we use fundamentalists and three different types of contrarians. This situation is not a usual one to normal circumstances on the market hence the results are remarkable.

Table 2
Parameters of the System for Case 2

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = -1.1$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = -0.3$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = -0.5$	$b_4 = 0$	Contrarians

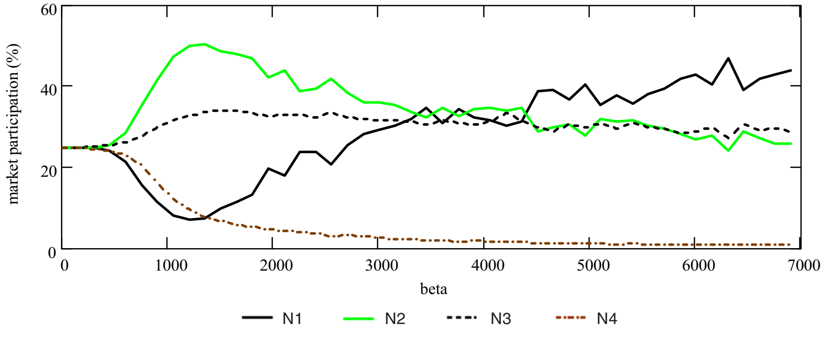
Without memory the system has complicated dynamics with maximum values of x within the interval ⟨-0.5, +0.7⟩. The role of fundamentalists N1 and contrarians without bias N4 is, with rising β, negligible (see Figure 4).

Figure 4
Participation of Trading Strategies on the Market with the Constant Memory Length (m = 1 for N1, N2, N3, N4, and different values of the parameter β)



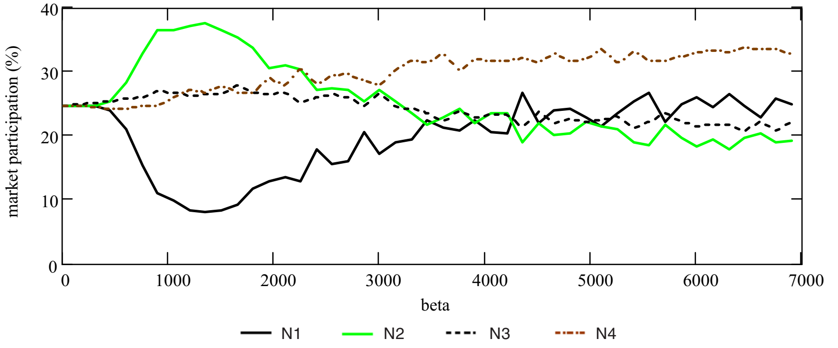
With longer memory (m = 20), the system is more stable, the price is less volatile, and the amplitude is smaller. With higher β (> 4300) the strategy of fundamentalists N1 becomes the most profitable strategy on the market. From the beginning, contrarians, without bias N4, lose their positions and almost diminish from the market (see Figure 5). With such memory, it is evident the importance of bias for contrarians N3 versus N4.

Figure 5
Participation of Trading Strategies on the Market with the Constant Memory Length (m = 20 for N1, N2, N3, N4, and different values of the parameter β)



Further analysis has shown the system sensitivity on the memory lengths. The following case has the same coefficients for trading strategies but with shorter memory length for pure contrarians N4. In this example, it is evident the increase of profitability of the strategy N4 and also for β > 2300 it becomes the most profitable one (see Figure 6).

Figure 6
Participation of Trading Strategies on the Market with the Constant Memory Length
(m = 20 for N1, N2, N3, the memory length m = 10 for N4, and different values of the parameter β)



Case 3: Fundamentalists, Trend Chasers and Contrarians

For the last case, consider system with the following parameters:

Table 3
Parameters of the System for Case 3

Type	Parameters		
N1	$g_1 = 0$	$b_1 = 0$	Fundamentalists
N2	$g_2 = 1.0$	$b_2 = 0.2$	Trend with upward bias
N3	$g_3 = 0.6$	$b_3 = -0.2$	Trend with downward bias
N4	$g_4 = -0.5$	$b_4 = 0$	Contrarians

For the memory-less model the leading strategies on the market are trend chasers with downward bias N3 (see Figure 7). With memory (m = 20), the contrarians N4 are becoming the leading strategy on the market. Strategies N1 and N3 are almost exiting the market (see Figure 8).

Figure 7
Participation of Trading Strategies on the Market with the Constant Memory Length
(m = 1 for N1, N2, N3, N4, and different values of the parameter β)

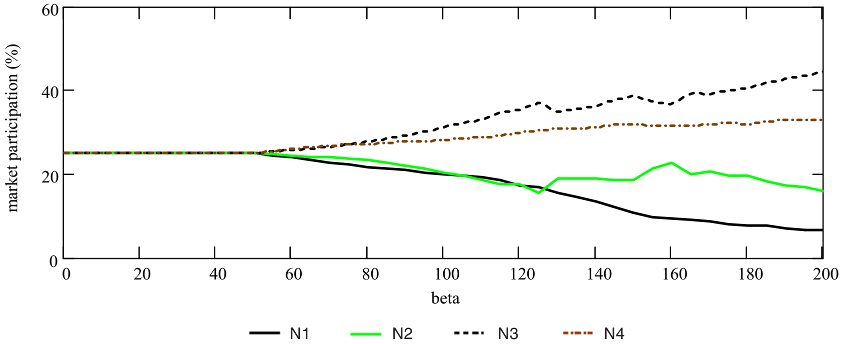
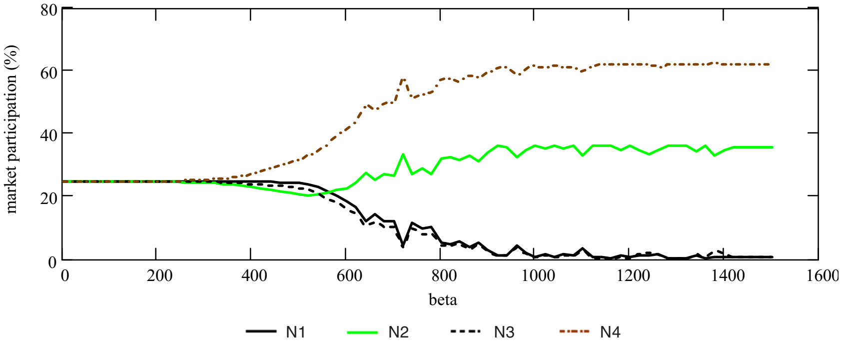
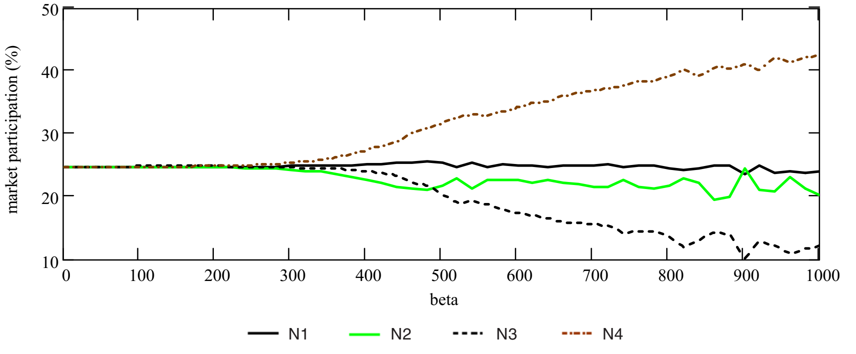


Figure 8
Participation of Trading Strategies on the Market with the Constant Memory Length
(m = 20 for N1, N2, N3, N4, and different values of the parameter β)



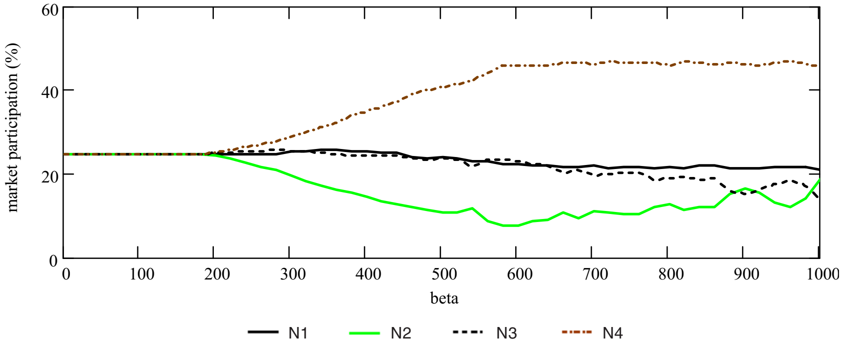
In this case, there also exists significant sensitivity on the memory length. Changing the memory length for fundamentalists N1 they are becoming the significant part of the market (see Figure 9).

Figure 9
Participation of Trading Strategies on the Market with the Constant Memory Length
(*m* = 20 for N2, N3, N4, the memory length *m* = 10 for N1, and different values of the parameter β)



Moreover, using the memory reduction not only for strategy N1 but also for strategy N3 (the least profitable strategy) we get similar result for fundamentalist N1 as in the preceding case. The dominance of contrarians N4 is the same as in the preceding case. However, due to the memory reduction for trading strategy N3 their profitability is remarkably increased (see Figure 10).

Figure 10
Participation of Trading Strategies on the Market with the Constant Memory Length
(*m* = 20 for N2, N4, the memory length *m* = 10 for N1, N3, and different values of the parameter β)



6. Conclusion

The system with memory is more stable than the memory-less system. Paradoxically, higher values of β are needed to generate chaotic behaviour.

In all cases, memory adding helps fundamentalists to increase profit, i.e., to increase participation on the market. Especially in the first case and in the second case they even become the most profitable strategy as β increases. That is a remarkable difference with comparison to the memory-less system.

In cases 2 and 3 we have shown a dependence of the profitability strategies on different memory lengths within the system. In case 2, a fact of shorter memory length for pure contrarians N4 changes a profitability of this strategy significantly. From the marginal participation on the market, this strategy is becoming the leading strategy as β increases. In case 3, the memory reduction for N1 and N3 helps increase profits to these strategies also.

It is shown that increased memory helps contrarians outperform other strategies on the market.

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