

Abstract

This study questions the importance of accounting for sectoral heterogeneity in a DSGE model for the Czech Republic. The benchmark DSGE model originally developed by the Czech Ministry of Finance benefits from features such as wage and price stickiness, habit formation in the utility function and capital adjustment costs. The Input-Output DSGE model extended hereby proves to provide more precise estimates for the evolution of aggregate variables and to supply a more detailed structure of the economy. The set of variables the dynamics of which significantly improve consists of inflation rate and nominal interest rate. The disaggregated model also fits data well in terms of sectoral production functions. Finally, the absence of industrial heterogeneity in the model is shown to lead to an underestimation of the impact of the technology shock on the Czech gross domestic product.

Keywords: Czech Republic, DSGE, Input-Output DSGE model

JEL Classification: C67, C68, E17

1. Introduction

Dynamic Stochastic General Equilibrium models belong amongst the most common tools for economic forecasting (see, for example, Ambriřko *et al.*, 2012; Christiano *et al.*, 2005; Smets and Wouters, 2007; Aliyev and Bobková, 2014). This study adds to the research into DSGE models by introducing reality-of-product differentiation stemming from sectoral heterogeneity.

Sectoral differentiation stems from different patterns of firms' behaviour across industries. Intermediate products flow between these sectors based on demand of individual firms operating in an industry. With such a tool, monetary policy-makers can target sectoral inflation rates as advocated by Aoki (2001), Benigno (2004) and Huang and Liu (2004).

The most influential papers on the topic of sectoral heterogeneity are Bouakez *et al.* (2009a, 2009b) and Bisová *et al.* (2013). In contrast to Bouakez *et al.* (2009a, 2009b), the present study also outlines derivation of the New Keynesian Phillips curve for price and wage selection behaviour subject not only to monetary policy shocks but also to fiscal policy shocks. Opposite to the model constructed herein, the model of Bisová *et al.* (2013) assumes a symmetric input-output case and estimates a rather simplified RBC model.

The core model subject to disaggregation in this study has been designed by the Czech Ministry of Finance (See Aliyev and Bobková, 2014). The advantage of this model is its

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relatively clear composition and New Keynesian character with commonly applied features such as wage and price stickiness, capital adjustment costs or habit persistence formation. The industries included in the extended benchmark model are the industrial and financial sectors and a sector consisting of all the remaining products not produced in the previous sectors. Despite this simplification, the model can be subject to any degree of sectoral disaggregation.

The presented model (Input-Output DSGE model or IO DSGE model) includes sectors as heterogeneous in production functions which differ in their combination and weight of intermediate inputs, capital and labour demands. The forms of the sectoral production functions assimilate the aggregate one; they are represented as Cobb-Douglas production functions. The presumption of substitution character in Cobb-Douglas production functions between individual inputs represents one of the limitations of this production-function form. While capital and labour are commonly assumed as mutual substitutes to some degree, the same does not need to be true for individual intermediate inputs. A possible way to overcome this issue is to assume constant elasticity of the substitution production function or its even more advantageous nested CES.

Nevertheless, an advantage of the Cobb-Douglas type of production function is its fairly close correspondence with a real production process as well as its algebraic tractability. The production process in the benchmark model also takes the form of the Cobb-Douglas production function. In this article, I assume the traditional Cobb-Douglas production function to enable clear comparison of the extended IO DSGE model with its original counterpart.

Next, besides taking into account the input-output flow between sectors and differentiated price-setting behaviour for these intermediate inputs, the model reflects heterogeneous labour demand based on differences in wages and varying capital demand with different rental rates and utilization rates of capital across individual sectors.

The shocks analysed in this study are the technology and the monetary policy shock. The individual variables display the same direction of reaction to these shocks alike the outcomes of other textbook New Keynesian models (see Galí, 2015; McCandless, 2008). Finally, the comparison of selected nominal variables for the original model and for the extended version with real data displays closer correspondence of the latter model to reality.

In regard to individual sectors, the simulation results display high sensitivity of the industrial sector to the monetary policy shock, which corresponds to the study by Gawthorpe and Safr (2015). Caraiani (2009) as well as Bouakez *et al.* (2009b) get similar findings regarding high vulnerability of the construction sector towards the monetary policy shock.

The paper is organized as follows. The next part provides a mathematical representation of the model. The subsequent section explains data for estimated parameters of the Czech economy. A discussion of the simulated results for the technology and the monetary policy shock follows with a comparison of the extended model with its original version. Conclusion summing up the findings can be found at the end of this paper.

2. The Input-Output DSGE Model

The derivation of the model in this study follows the benchmark model (see Aliyev and Bobková, 2014). The IO DSGE model incorporates all features such as heterogeneity of households, wage and price stickiness, capital adjustment costs or utilization rate for capital. Beyond the original model, the extended version disaggregates the production function to account for sectoral heterogeneity in input demands as well as input prices.

The main disparity between the two models concerns the Firm and the Labour sections with a derivation of the New Keynesian Phillips curves. The production in the economy is divided into three sectors. The Industrial sector, the Financial sector and the Other sector, which consists of products not produced by the preceding sectors. The inclusion of only three sectors simplifies the model simulation. Nevertheless, the model formulation allows any degree of disaggregation. The studies of Bouakez *et al.* (2009a, 2009b) motivate the selection of the Industrial sector as it appears in their study as very sensitive to the monetary policy shock. The Financial sector is also expected to be very susceptible to restrictive monetary policy as the interest rate is the price for its product – loans.

2.1 Households

This section outlines the household sector, which stays identical to the benchmark model (see Aliyev and Bobková, 2014). Heterogeneity of households indexed as $j \in [0, 1]$ is the first major assumption, where only Savers follow habit persistence. Households of the Spenders type do not decide on their expenditures as they spend their entire income on consumption.

The dynamic expression of the model dealing with the optimization problem specifies the individual of the “Saver” type indexed as R as one choosing a time path over consumption and labour supply. The model assumes a continuum of identical and infinitely-lived households. Individuals’ preferences are given by an infinite stream of utility in the following form:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\log(C_{j,t+n}^R - H_{j,t+n}) - \frac{(N_{j,t+n}^R)^{1+\psi^N}}{1+\psi^N} \right], \quad (1)$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor and beta equals $\frac{1}{1+\Omega}$, where omega represents the subjective rate of time preference. The consumption variable is labelled $C_{j,t}$ and $H_{j,t} = h_r C_{j,t-1}$ stands for the external habit formation, where h_r is the habit formation parameter which determines the dependence of present consumption on its past level. $N_{j,t}^R$ denotes labour supply of Savers. ψ^N is a parameter measuring the real wage elasticity of labour supply.

The aggregate budget constraint for Savers takes the following form:

$$\begin{aligned} (1 + \tau_t^c) P_t^C C_{j,t}^R + P_t^I I_{j,t} + P_t^a(u_{j,t}) K_{j,t}^S + \frac{1}{R_t} B_{j,t+1} + \frac{1}{R_t^* + \zeta_t} S_t B_{j,t+1}^* = B_{j,t} + S_t B_{j,t}^* + \\ + (1 - \tau_t^k) [R_t^k u_{j,t} K_{j,t}^S + Q_t] + (1 - \tau_t^w) W_t N_{j,t}^R. \end{aligned} \quad (2)$$

The left-hand side of the equation starts with the term for consumption expenditure $P_t^C C_{j,t}^R$ multiplied by the consumption tax τ_t^c , followed by the investment $I_{j,t}$ with the investment price P_t^I . The capital variable $K_{j,t}^s$ faces the capital adjustment costs $a(u_{j,t})$, which is an increasing convex function. Next, Saver households decide on investment in domestic $B_{j,t}$ and foreign bonds $B_{(j,t)}^*$ where the latter ones are denominated in the Czech crowns with the help of the nominal exchange rate S_t . R_t and R_t^* stand for the domestic and the foreign interest rate respectively. Premium on foreign bonds is labelled as ζ_t .

The decision on the households' expenditure is dependent on their wealth summarized by the right-hand side of the equation. In addition to wages W_t reduced for the income tax τ_t^w , households earn capital rents $R_t^k u_{j,t} K_{j,t}^s + Q_t$ and profits Q_t from ownership of firms, which decrease for the corporate tax rate τ_t^k . These tax rates labelled with the Greek letters tau enter the model as simple autoregressive AR(1) processes. Concerning capital stock $K_{j,t}^s$, $u_{j,t} = K_{j,t}^s / K_{j,t}^s$ is the utilization rate of the capital.

The capital stock $K_{j,t}^s$ evolves as

$$K_{j,t+1}^s = (1 - \delta) K_{j,t}^s + \left[1 - \Upsilon \left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \quad (3)$$

where δ denotes the depreciation rate.

and $\Upsilon \left(\frac{I_t}{I_{t-1}} \right) \equiv \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - u_z \right)^2$ is the functional form which expresses the capital adjustment term. The parameter κ denotes capital adjustment costs and u_z is a steady-state growth rate of technology.

This latter adjustment cost variable in the capital stock equation follows the intuition of permanent growth for some real variables such as consumption, capital or investment. To detrend the non-stationary variables, these variables are assumed to follow a permanent technology shock z_t (see Aliyev and Bobková, 2014). The stochastic technology trend is then given by:

$$u_{z,t} = \frac{z_t}{z_{t-1}}, \quad (4)$$

where $u_{z,t}$ is the growth rate of technology, which equals an autoregressive process in the form:

$$u_{z,t} = (1 - \rho_{uz}) u_z + \rho_{uz} u_{z,t-1} + e_t^{uz}. \quad (5)$$

Those stationarized variables which previously had an apparent trend on data will be labelled with small letters.

Furthermore, in this model the second type of households is called Spenders. The Spender type of households does not optimize. Their expenditure is fully determined by their income and their labour supply is inelastic.

The detrended evolution of consumption for Spenders labelled N corresponds to the budget constraint:

$$(1 + \tau_t^c) P_t^c c_t^N = \frac{1}{z_t} \left\{ (1 - \tau_t^w) W_t N_{j,t}^N + \tau_t^b W_b (N_{j,t}^N - L_{j,t}^N) + TR_t \right\}. \quad (6)$$

The letters stand for the same variables as in the budget constraint for Savers. The TR_t labels transfers, W_b wage bill and τ_t^b unemployment benefits, $N_{j,t}^N - L_{j,t}^N$ labels the difference between labour supply and labour demand, which expresses unemployment in this model. The variables with small letters are also detrended with the help of the stochastic technology trend z_t . Those stationarized variables are expressed by small letters. The derivation of the optimal conditions for both types of households stay identical to the benchmark version. Interested readers might find a detailed description in Aliyev and Bobková (2014). What differs from the original model is the firm sector, which is described in detail in the following section.

2.2 Firms

The continuum of firms in this model is indexed as $i \in [0, 1]$. Every firm has monopolistically competitive power and thus products of these firms are differentiated. Additionally, there is heterogeneity of firms across sectors. Firms from different sectors then vary not only in the differentiation of their goods but also by their production functions with various degrees of employment of individual inputs.

The sectoral production function in real terms is:

$$Y_t^j(i) = (K_t^j)^{\alpha(j)} (M_t^j)^{\beta(j)} (Z_t L_t^j)^{1-\alpha(j)-\beta(j)}, \quad (7)$$

where Z_t stands for the level of technology exogenously evolving over time. The parameters $\alpha(j)$ and $\beta(j)$ are the output elasticities of capital and intermediate input, respectively. The term $1 - \alpha(j) - \beta(j)$ expresses the output elasticity of labour. The variables L_t^j and K_t^j stand for the labour demand and capital demand in the sector j respectively. The variable M_t^j is a bundle of differentiated intermediate inputs defined as the Dixit-Stiglitz index (see Dixit and Stiglitz, 1977). This variable M_t^j illustrates material inputs flowing into the sector j :

$$M_t^j = \left(\sum_{i=1}^N \chi_{ji} (M_{i,t}^j)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \text{where} \quad \sum_{i=1}^N \chi_{ij} = 1 \quad (8)$$

and j denotes the sector in which the particular firm i produces its output. This composite index M_t^j displays the way in which material inputs are combined in the sector j . It assimilates the consumption index with a difference of the new parameter $\chi_{i,j}$. This parameter stands for the weight the input receives in the sector j . The price of the composite good M_t^j labelled as H_t^j is:

$$H_t^j = \left(\sum_{i=1}^N (\chi_{ji})^{\theta} (H_{i,t}^j)^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad (9)$$

which appears identical to Bouakez *et al.* (2009b, p. 6).

To derive the optimal input demands in a detrended form, one needs to stationarize values for capital and intermediate-input demands, as these two variables follow a clear

trend in the Czech data. Formally, multiplication of the non-stationary input variables by the trend Z_t results in the production function:

$$y_t^j(i) = Z_t (k_t^j)^{\alpha(j)} (m_t^j)^{\beta(j)} (L_t^j)^{1-\alpha(j)-\beta(j)}. \quad (10)$$

Maximizing this production function in respect to the detrended total cost function:

$$c_t^j(i) = R_t^{k,j} k_t^j + W_t^j L_t^j + H_t^j m_t^j \quad (11)$$

and taking into account the intermediate-inputs bundle yields the material input demand for the sector j :

$$m_{i,t}^{j*}(i) = \omega_M \left(y_t^j \frac{(H_t^j)^{\beta(j)} (R_t^k)^{\alpha(j)} (W_t^j)^{1-\alpha(j)-\beta(j)}}{(Z_t)^{1-\alpha(j)-\beta(j)} (m_t^j)^2} \frac{\chi_{ji}}{H_{i,t}^j} \right)^\theta, \quad (12)$$

where $\omega_M = \frac{\beta(j)}{1-\alpha(j)-\beta(j)} \frac{\frac{1-2\beta(j)}{1-\beta(j)} 1-\alpha(j)-\beta(j)}{\alpha(j)} \frac{\alpha(j)-\alpha(j)\beta(j)}{1-\beta(j)}$

Maximization of the same production function with respect to the total costs also results in the optimal labour and capital demands for the sector j :

$$L_t^{j*}(i) = \omega_L y_t^j(i) (H_t^j)^{\beta(j)} (R_t^k)^{\alpha(j)} Z_t^{-1} (W_t^j)^{\left(\frac{(1-\alpha(j)-\beta(j))\beta(j)+\alpha(j)}{1-\beta(j)} \right)}, \quad (13)$$

where $\omega_L = \left(\frac{1-\alpha(j)-\beta(j)}{\alpha(j)} \right)^{\frac{\alpha(j)-\alpha(j)\beta(j)}{1-\beta(j)}} \left(\frac{1-\alpha(j)-\beta(j)}{\beta(j)} \right)^{\beta(j)}$,

$$k_t^{j*}(i) = \omega_K y_t^j(i) (H_t^j)^{\beta(j)} W_t^{1-\alpha(j)-\beta(j)} Z_t^{-1} (R_t^k)^{\left(\frac{\alpha(j)\beta(j)+(1-\alpha(j)-\beta(j))}{1-\beta(j)} \right)}, \quad (14)$$

where $\omega_K = \left(\frac{1-\alpha(j)-\beta(j)}{\beta(j)} \right)^{\beta(j)} \left(\frac{\alpha(j)}{1-\alpha(j)-\beta(j)} \right)^{\frac{1-\alpha(j)-\beta(j)+\alpha(j)\beta(j)}{1-\beta(j)}}$.

Aggregation of the labour demand then also follows the CES Dixit and Stiglitz (1977) index:

$$L_t = \left[\sum_{i=1}^N \zeta^j \left(L_t^j \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (15)$$

where ζ^j is the share parameter.

Similarly, aggregation of the capital demand is:

$$K_t = \left[\sum_{i=1}^N \zeta^j \left(K_t^j \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (16)$$

with the share parameter ζ^j .

Next, this model, like the benchmark version, assumes monopolistic competition and a staggered price setting where only a fraction $1 - \rho$ of firms can reset their price optimally at the time t , while the rest of them sticks prices to lagged inflation.

The next part of this study describes price-setting behaviour for final and intermediate products produced with the above derived optimal combination of inputs. The price-selection procedure is in accordance with McCandless (2008).

2.2.1 Optimal price setting

A firm i from a sector j re-optimizing in period t selects the price $P_t^*(i)$ that maximizes the market value of the profits while that price remains effective. $(1 - \rho)$ labels the fraction of firms that is able to adjust their price in the period t . The price of final products is assumed to follow the same pattern in all sectors. The final goods-producing firm transforms goods from other firms into homogeneous goods:

$$Y_t = \left[\sum_{i=1}^N \left(Y_{i,t} \right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad (17)$$

where the parameter for the constant elasticity of substitution function defined above is $\nu > 1$.

The firm solves a profit maximization problem of the form:

$$\max \sum_{k=0}^{\infty} (\beta \rho)^k E_t \{ P_{i,t+k}^* Y_{i,t+k} - P_{t+k} MC_{t+k} Y_{i,t+k} \}, \quad (18)$$

where MC_{t+k} stands for aggregate marginal costs. The parameter ρ labels the price stickiness in the model. Substitution of the individual demand for the product i , $Y_{i,t+k}$, which is differentiated across firms:

$$Y_{i,t+k} = \left(\frac{P_{t+k}}{P_t^*} \right)^{\nu} Y_{t+k} \quad (19)$$

into the profit maximization problem yields the optimal price setting behaviour for the firm i in the time t

$$\hat{P}_t^*(i) = \frac{\rho}{1 + \beta \rho^2 u_z} \hat{P}_{i,t-1} + \frac{\beta \rho u_z}{1 + \beta \rho^2 u_z} \hat{P}_{i,t+1} + \frac{(1 - \rho)(1 - \beta \rho u_z)}{1 + \beta \rho^2 u_z} \widehat{MC}_t. \quad (20)$$

The hats signal log-linearized notation. The derivation of the marginal costs function follows the modification with detrended capital and intermediate-input demand variables.

$$\widehat{MC}_t = \beta \hat{H}_t + \alpha \hat{R}_t^k + (1 - \alpha - \beta) \hat{W}_t - \hat{Z}_t. \quad (21)$$

In addition to the price-setting behaviour for final goods, individual firms also decide on their prices for intermediate products. These firms in the sector j are assumed to set the same price on their intermediate products regardless which sector demands it.

The existence of monopolistic competition also at this level motivates the firms to base their choice of prices for intermediate products with regard to the rigidity of prices and thus to follow a similar pattern as that for prices of final products.

In a log-linearized form the, equations for the intermediate input prices in the sector j are:

$$\hat{H}_{i,t}^j = \frac{\rho}{1 + \beta \rho^2 u_z} \hat{H}_{i,t-1}^j + \frac{\beta \rho u_z}{1 + \beta \rho^2 u_z} \hat{H}_{i,t+1}^j + \frac{(1 - \rho)(1 - \beta \rho u_z)}{1 + \beta \rho^2 u_z} \widehat{MC}_t^j, \quad (22)$$

where

$$\widehat{MC}_t^j = \alpha \hat{R}_t^{kj} + (1 - \alpha - \beta) \hat{W}_t^j - \hat{Z}_t. \quad (23)$$

In the marginal costs equation, \hat{R}_t^{kj} stands for the real rental rate of capital as derived from the optimization problem of households. The log-linearized version of the FOC for the rental rate of capital for every sector is:

$$\hat{R}_t^{kj} = \frac{a''(\bar{u})}{a'(\bar{u})} \hat{u}_t^j + \frac{\bar{\tau}^k}{1 - \bar{\tau}^k} \hat{\tau}_t^k + \hat{P}_t, \quad (24)$$

where $\hat{u}_t^j = \hat{k}_t^j - \hat{k}_t^s + \hat{u}_{z,t}$.

The real rental rate of capital in an aggregate form is:

$$R_t^k = \left(\sum_{i=1}^N (\varsigma^j)^\theta (R_t^{kj})^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad (25)$$

with the share parameter ς^j . The next section outlines the optimal wage-setting behavior for sectoral wages \hat{W}_t^j . The derivation of particular equations assimilates the original version (see Aliyev and Bobková, 2014).

2.3 Optimal wage setting

Each household i is a monopoly supplier of labour. Labour is not mobile across sectors in this model. The labour bundle N_t is composed of heterogeneous labour N_t^j which differs across sectors and within sectors respectively:

$$N_t = \left[\sum_{i=1}^N \varsigma^j \left(N_t^j \right)^{\frac{\varepsilon+1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon+1}}, \quad (26)$$

where the share parameter in the above CES function is ς^j .

$$N_t^j = \left[\sum_{i=1}^N \left(N_t^{i,j} \right)^{\frac{\theta_w-1}{\theta_w}} \right]^{\frac{\theta_w}{\theta_w-1}}. \quad (27)$$

The representative household then sets wages in the period t in the sector j denoted as W_t^{j*} to maximize its utility function in respect to the budget constraints while considering the labour demand relationship:

$$N_t^{i,j} = N_t^j \left(\frac{W_t^j}{W_t^{j*}} \right)^{\theta_w}. \quad (28)$$

The wage aggregation equation stays as in the original version:

$$\left(W_t^j \right)^{1-\theta_w} = \alpha_R \left[\epsilon_w \left(W_{t-1}^j \right)^{1-\theta_w} + (1 - \epsilon_w) \left(W_t^{j*} \right)^{1-\theta_w} \right] + (1 - \alpha_R) \left(W_{t-1}^j \right)^{1-\theta_w}. \quad (29)$$

Manipulation of the resulting first-order condition leads to the final stationarized wage equation for the sector j in the model:

$$p_{ww} \hat{W}_t^{j*} = \beta \epsilon_w \hat{W}_{t+1}^j + p_{wi} \hat{W}_{t-1}^j + p_{wl} E(\hat{L}_t^j) - p_{wn} \hat{N}_t^j \quad (30)$$

with

$$p_{ww} = \left\{ 1 + [1 - \alpha_R (1 - \epsilon_w)] \beta \epsilon_w \left(\frac{1}{\bar{\pi}_w} \right)^{1-\theta_w} + (1 - \beta \epsilon_w) \left[\frac{\theta^w}{\epsilon} - \alpha_R (1 - \epsilon_w) \left(\frac{\theta^w}{\epsilon} + 1 \right) \right] \right\},$$

$$p_{wi} = \left\{ [1 - \alpha_R (1 - \epsilon_w)] \left(\frac{1}{\bar{\pi}_w} \right)^{1-\theta_w} + \frac{\theta^w}{\epsilon} [1 - \alpha_R (1 - \epsilon_w) (1 - \beta \epsilon_w)] \right\},$$

$$p_{wl} = \alpha_R \frac{1}{\epsilon} (1 - \epsilon_w) (1 - \beta \epsilon_w) \psi^N,$$

$$p_{wn} = \alpha_R \frac{1}{\epsilon} (1 - \epsilon_w) (1 - \beta \epsilon_w) \frac{\bar{N}}{\bar{N}^R},$$

where the sectoral wage can be aggregated as:

$$W_t = \left(\sum_{i=1}^N (\xi^j)^\theta (W_t^j)^{1-\theta} \right)^{\frac{1}{1-\theta}}. \quad (31)$$

The final wage equation 30 approximates the one from the original model. The growth of labour demand relatively to labour supply has inflationary impact on wages. Unemployment is negatively linked to wages, which stays in line with other NKE models such as that of Galí (2015).

2.4 Foreign sector

This sector stays unaltered from the benchmark model. The Czech economy is a small open economy where the calculation of imports follows:

$$MM_t = C_t^M + I_t^M \quad (32)$$

and the export equation is:

$$X_t = C_t^{M*} + I_t^{M*}. \quad (33)$$

Foreign consumption and investment are assumed to evolve like the domestic consumption and investment, respectively. To denominate prices in the same Czech currency, the nominal exchange rate has to be derived:

$$S_t = \frac{P_t^M}{P_t^*}. \quad (34)$$

The uncovered interest rate parity then combines the FOCs for foreign and domestic bonds:

$$\frac{E_t(S_{t+1})}{S_t} = \frac{R_t}{(R_t^* + \varsigma_t)}. \quad (35)$$

2.5 Monetary policy

The monetary policy rule assimilates the original version extended for the monetary policy shock:

$$\hat{R}_t = (1 - \phi_r) [\lambda_\pi \hat{\pi}_t + \lambda_y \hat{y}_t] + \phi_r \hat{R}_{t-1} + v_t, \quad (36)$$

The central bank is assumed to target inflation. The parameter ϕ_r reflects the persistence of the interest rate and the parameters λ_π , λ_y the responsiveness of the central institution to inflation and output, respectively.

2.6 Shocks

The economy is subject to the following exogenous shocks. The exogenous component of the interest rate is assumed to follow an AR(1) process:

$$v_t = \rho_v v_{t-1} + e_t^v, \quad (37)$$

where $\rho_v \in (0, 1)$ and e_t^v is a zero-mean white-noise process.

An expansionary monetary policy is connected with a negative realization of e_t^v .

Other shocks introduced also follow the autoregressive process. Namely, the shock into the foreign prices:

$$\hat{P}_t^* = \rho_{p^*} \hat{P}_{t-1}^* + e_t^{p^*}. \quad (38)$$

Shock into the capital tax:

$$\hat{\tau}_t^k = \rho_{tk} \hat{\tau}_{t-1}^k + e_t^{tk}. \quad (39)$$

Shock for the tax on consumption:

$$\hat{\tau}_t^c = \rho_{tc} \hat{\tau}_{t-1}^c + e_t^{tc}. \quad (40)$$

Foreign interest rate shock:

$$\hat{R}_t^* = \rho_R \hat{R}_{t-1}^* + e_t^R. \quad (41)$$

Shock into the income tax:

$$\hat{\tau}_t^w = \rho_{tw} \hat{\tau}_{t-1}^w + e_t^{tw}. \quad (42)$$

Shock into the transfers from the government

$$\hat{tr}_t = \rho_r \hat{tr}_{t-1} + e_t^{tr}. \quad (43)$$

The next section describes the estimation of parameters used for the simulation of the log-linearized model.

3. Parameter Estimates

Table 2 in the Appendix presents the parameter estimates for this model. Parameters for the aggregate (not sectoral) variables stay identical to those in Aliyev and Bobková (2014).

Sectoral parameters are estimated for the three sectors of interest. The indexes for these sectors are 1, 2 and 3, respectively. The first/industrial sector incorporates production of commodities marked by CZ-CPA 05-33 and the second/financial sector consists of goods or services indicated by CZ-CPA 64-66. The third sector comprises all the other firms producing other goods/services than the previous sectors.

Input-output matrices from the Czech Statistical Office contain the necessary data for the estimation procedure of the IO parameters such as for the weight parameters in the composite indexes ξ^j , χ_{ij}^j , χ_j and ζ^j . The parameter ξ^j denotes the fraction of capital in the sector j in the total capital. The parameters χ_{ij}^j stand for the share of intermediate input flowing from the sector i to the sector j in the total demand for intermediate inputs of the sector j and χ_j labels the share of intermediate input in the sector j in the total intermediate input demand in the economy. Finally, the parameter ζ^j reflects the share of labour in the sector j in the aggregate labour variable in the Czech economy, where labour stands for hours worked.

The weights stand for the share of the sectoral variables in the aggregate variables. This calculation procedure stays in line with other authors such as Bouakez (2009b). The input-output matrices are available with a time span of five years on the website of the Czech Statistical Office (www.czso.cz). The year selected for the parameter estimation is 2015. The input-output table contains 6724 observations for over 90 industries. Table 1 in the Appendix provides the summary statistics for the input-output matrix.

Sectoral parameters for individual production functions $\alpha(j)$, $\beta(j)$ were estimated using the vector autoregressive model method based on data from 1995 to 2014 available from the Czech Statistical Office (see www.czso.cz). The databases of national accounts present the variables in the production function in a yearly frequency. The production function was modified to the subsequent form for the estimation procedure:

$$y_t^j / L_t^j = Z_t \left(k_t^j / L_t^j \right)^{\alpha(j)} \left(m_t^j / L_t^j \right)^{\beta(j)}, \quad (44)$$

where the individual variables of output, capital and intermediate input are divided by labour. The estimation of the above equation with the VAR regression method is in a logarithmic form:

$$\widetilde{y}_t^j = \alpha(j) \widetilde{k}_t^j + \beta(j) \widetilde{m}_t^j + \widetilde{\varepsilon}_t^j. \quad (45)$$

4. Discussion of the Results

This section provides an analysis of the shocks for the extended version of the benchmark model. The graphical representation of the evolution of individual variables is presented in Figures 1 and 2 in the Appendix. In the charts for the aggregate variables, the dashed line depicts the effect of the shock on the original model and the solid line on the extended input-output version. To examine the three sectors of interest, the charts titled m_j , l_j , k_j illustrate the sectoral inputs followed by labour and capital demands. The symbol “ m_{ij} ” stands for the input-output flows from the sector i to the sector j . For these sectoral variables, the solid line depicts the first sector, the dashed line the second sector and finally the dotted line the third sector.

4.1 Technology shock

The results for the technology shock are available in Figure 1 in the Appendix. This shock enters the model in the form of realization e_t^{tz} size 0.1. The direction of reaction for the aggregate variables towards the shock assimilates other traditional New Keynesian DSGE models (see Galí, 2015; McCandless, 2008). In line with the results of Galí (2015) or Bordo *et al.* (2004), the technology shock booms productivity. Subsequently, a sudden increase in money demand in response to the productivity growth has a deflationary effect on prices, pulling down inflation. Input prices follow such a deflationary pattern. Production-input demands also drop as individual companies demand less of now more productive inputs. To stabilize the economy, the central bank follows the Taylor rule by reducing the nominal interest rate. This direction of the technology shock impact assimilates the model of Galí (2015).

This shock has a permanent effect on the selected variables. This effect stems from the introduction of the unit-root technology shock into the model as designed in the benchmark Ministry of Finance model. The third sector's productivity rises the most in response to the technology shock, followed by the first and then by the second sector.

To explain the most significant reaction of the third sector's production, it is necessary to outline its composition. This sector consists of all other goods not produced in the remaining sectors. Besides agricultural products, it consists mostly of services such as education, research, social security, defence, accommodation or cultural activities. As Bouakez *et al.* (2009b) illustrates, services commonly sell for sticky prices. The monopolistically competitive producers then accommodate the increase in aggregate demand by raising its product.

The second sector most susceptible to this shock is the industrial sector. The high susceptibility of the industrial sector to the shock corresponds to the previous study (see Gawthorpe and Safr, 2015). The industrial sector includes mining, construction but also non-durable manufacturing which face more flexible prices (see Bouakez *et al.*, 2009b). This sector is then affected by the increase in aggregate demand indirectly, through the input-output structure. In addition, the flexible prices of this sector can partially accommodate the shock. In sum, while the technology shock affects both the third and the second sectors, the increase in aggregate demand has a more significant impact on the third sector.

In regard to labour and capital, demands for these inputs support the pattern of output increases, where the most productive third sector requires the least inputs as it employs them most efficiently. The first sector faces the second most sizeable decrease in these input demands and finally the second sector's demand for these inputs decreases the least.

The intermediate input flows between sectors follow subsequent sequence. The lowest decrease in intermediate inputs is apparent in the third sector, whose demands for inputs from the second, the first but also the third sector drop the least. The drop in intermediate inputs demanded by the first sector from all the other sectors is relatively more significant than that in the third sector. The financial sector demands the least of the inputs flowing from the first, the third but also from the second sector. Finally, the intuition behind the reduction

of the “ $m2j$ ”, or the input flow from the second sector to others, demonstrates lower necessity of money borrowings in times of prosperity. This evidence then helps understand why the second sector is the least positively affected by the shock in terms of output.

Altogether, the Input-Output extension of the original model exhibits a significant heterogeneity across sectors in reaction to the technology shock. Neglecting the sectoral differences can therefore bias the aggregate simulated evolution of the economy facing this type of shock.

4.2 Monetary policy shock

The restrictive monetary policy takes place in the form of realization size 0.25. The dynamics of individual variables in reaction to the shock assimilate the original model as well as the version of Galí (2015). The restrictive monetary policy negatively affects output as the loans become more expensive. The higher costs of investment result in lower productivity in the economy. The response appears to be different for individual sectors.

The asymmetric reaction to the shock across the sectors assimilates the output from the study of Bouakez *et al.* (2009b) and Peersman and Smets (2002), who estimated asymmetric response of sectors to the monetary policy shock in the United States and the Euro Area in general.

The output of the financial sector appears to be the most sensitive to the monetary policy shock followed by the product of the industrial sector. The remaining sector is the least sensitive to the monetary policy shock. The high vulnerability to the interest rate increases from the financial sector is intuitive. An increase in interest rate reduces demand for loans, which negatively affects balance sheets of banks. The simulation reveals the decrease in the demand for the input flows from the second sector to others as the most intensive one. Namely the Industrial, Financial and Remaining sectors lessen their demand from the Financial sector the most. This negative suppression of demand and, in turn, of profits for financial institutions creates a motive for the second sector to reduce its demand for intermediate inputs, capital and labour. Lower demand for all inputs helps fasten the transmission of the shock to the entire economy.

The second most sensitive sector to the shock is shown to be the Industrial sector. This outcome corresponds with our previous study (see Gawthorpe and Safr, 2015). The high sensitivity of the construction sector, which is included in the Industrial sector, is also a result found by Caraianni (2009) as well as Bouakez *et al.* (2009b). The profitability of the first industry is dependent on purchases of cheap loans. The growth of interest rate on loans as a consequence of this shock increases the total costs for the industrial sector. Subsequently, this sector reacts by decreasing production.

Finally, the intuition behind the sizeable reduction in product of individual sectors is the existence of the input-output flows among sectors. There exists a spillover effect where the presence of distress for one sector leads it to reduce input demands from others sectors. In detail, the financial sector drops its demand for the intermediate inputs more than other sectors. Such a reduction in demand negatively affects the remaining industries. The input-output structure therefore exacerbates the magnitude of the initial shock.

5. Comparison with the Benchmark Model

Figure 3 in the Appendix displays plots with selected variables to compare the Input-Output DSGE model with its symmetric benchmark version and the real data from the Czech Statistical Office for the time span from 1995 to 2014. The outlined variables include GDP and its individual components, inflation rate, nominal interest rate and sectoral production functions.

The gross domestic product variable evolves almost identically in the original DSGE model and the IO DSGE version. It also corresponds roughly close to the data. The graphical representation of the GDP components also appears very similar for both the benchmark model and the IO DSGE model.

What differs are the simulated data for inflation rate, nominal interest rate and sectoral production functions. All these variables approximate the reality better in the case of the IO DSGE model. Particularly, after 2005, the original model displays a tendency to overestimate inflation rate and nominal interest rate.

Overall, the evolution of the simulated IO DSGE model assimilates the benchmark model in terms of the selected aggregate real variables. However, the IO DSGE model is more precise in estimating the evolution of inflation rate, nominal interest rate and it also allows to account for the sectoral production functions.

6. Conclusion

This study aims to enhance the model by the Czech Ministry of Finance for individual sectors to question the significance of accounting for sectoral heterogeneity in a DSGE model.

It proves the ability of the DSGE model to fit data better if it is subject to such sectoral disaggregation. The heterogeneity of sectors then provides more detailed illustration of the economic behaviour in respect to the studied shocks.

Further research could provide more extensive disaggregation to the outlined model to account for higher variety of sectors. The present mathematical model is designed to account for any degree of sectoral disaggregation. A higher number of included sectors would allow us to capture the inter-sectoral flows in more detail but could also result in higher accuracy of the model outcome.

Next, the present model abstracts from accounting for sectoral variations in the household sector. The model could be extended to derive consumption expenditures for individual sectors as well as show investment flow into the sectors.

Finally, the constructed model could capture inter-regional flows in a way similar to how it describes inter-sectoral linkages. The number of equations in the script would grow exponentially with the number of regions, which would reduce the tractability of the model outcome. Next, such a regional model would have to account for inter-regional trade, which would necessitate a more precise disaggregated form of export and import equations. The popularity of regional models could then motivate researchers to use the present model with minor modifications to study inter-regional flows.

Appendix

Table 1 | Summary statistics

	Mean	Median	St.D.	Variance
X_{1i}	105,879	45,236	169,479	28,722,971,099
X_{2i}	51,444	51,435	22,866	52,285,402
X_{3i}	63,129	25,173	83,040	6,895,564,916
X_{11}	2,628	114	16,398	268,894,937
X_{12}	120	15	366	134,138
X_{13}	550	46	2,383	5,681,033
X_{21}	317	80	651	424,281
X_{22}	8,209	7,436	6,386	40,775,422
X_{23}	992	109	4,265	18,187,160
X_{31}	574	32	3,443	11,855,304
X_{32}	467	49	1,118	1,250,292
X_{33}	884	52	4,320	18,665,620

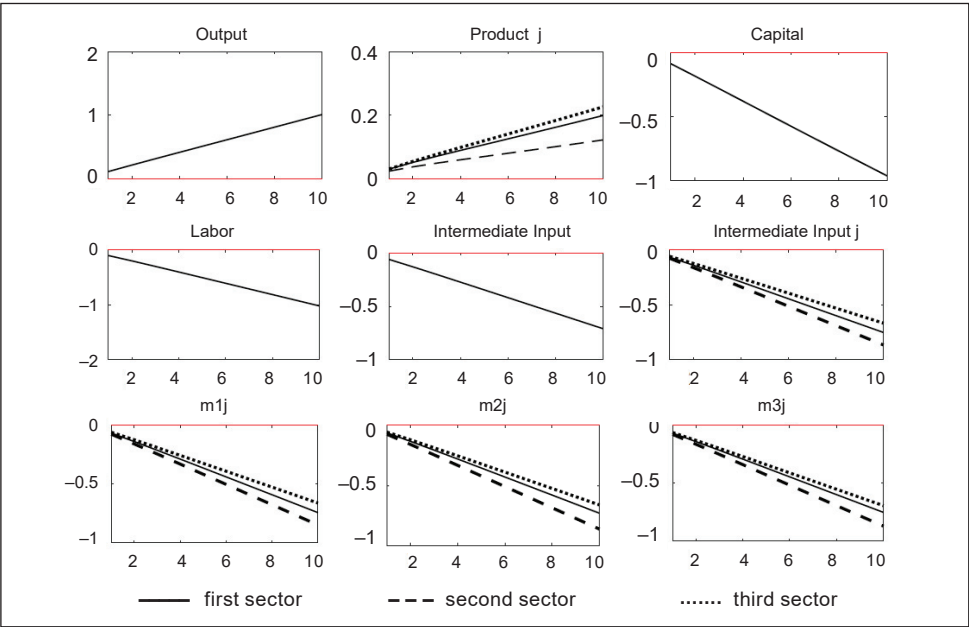
Source: Author's calculations. Data are available on the Czech Statistical Office website for 2015.

Table 2 | Parameter estimates

ρ_{a1}	0.2978	α_1	0.1757	ψ^N	1.55	X_{22}	0.4787	ϵ	0.2
ρ_{a2}	0.1569	α_2	0.0477	ω_{ss}	0.5	X_{31}	0.2969	ϵ_w	0.52
ρ_{a3}	0.4954	α_3	0.3187	ω_{si}	0.5	X_{32}	0.4536	φ_r	1.5
ρ_{p^*}	0.5	θ	2	ω_{snx}	0.25	X_{33}	0.8409	λ_π	1.5
ρ_R	0.52	θ_c	0.3	X_{1i}	0.4812	β	0.99	λ_γ	0.5
ρ_{tc}	0.6	θ_{c^*}	0.3	X_{2i}	0.0242	β_1	0.7867	ζ^1	0.2375
ρ_{tk}	0.91	θ_i	0.2	X_{3i}	0.4946	β_2	0.9036	ζ^2	0.0290
ρ_{tr}	0.92	θ_{j^*}	0.2	X_{11}	0.6944	β_3	0.6703	ζ^3	0.7335
ρ_{tw}	0.61	θ_w	2	X_{12}	0.0677	α_R	0.32	ξ^1	0.3047
ρ_{uz}	1.00	κ	11	X_{13}	0.1508	δ	0.01	ξ^2	0.0310
ρ_v	0.5	μ_{cm}	0.22	X_{21}	0.0087	μ_{im}	0.32	ξ^3	0.6642
X_{23}	0.0083	—	—	—	—	—	—	—	—

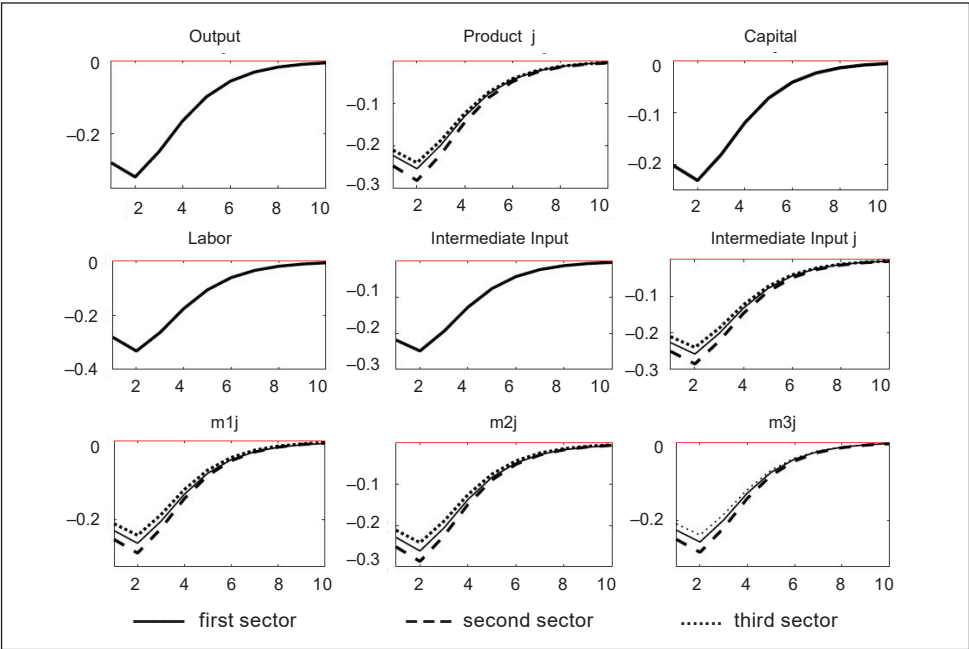
Source: Aliyev and Bobková, 2014, and author's own estimation based on data from the Czech Statistical Office.

Figure 1 | Technology shock



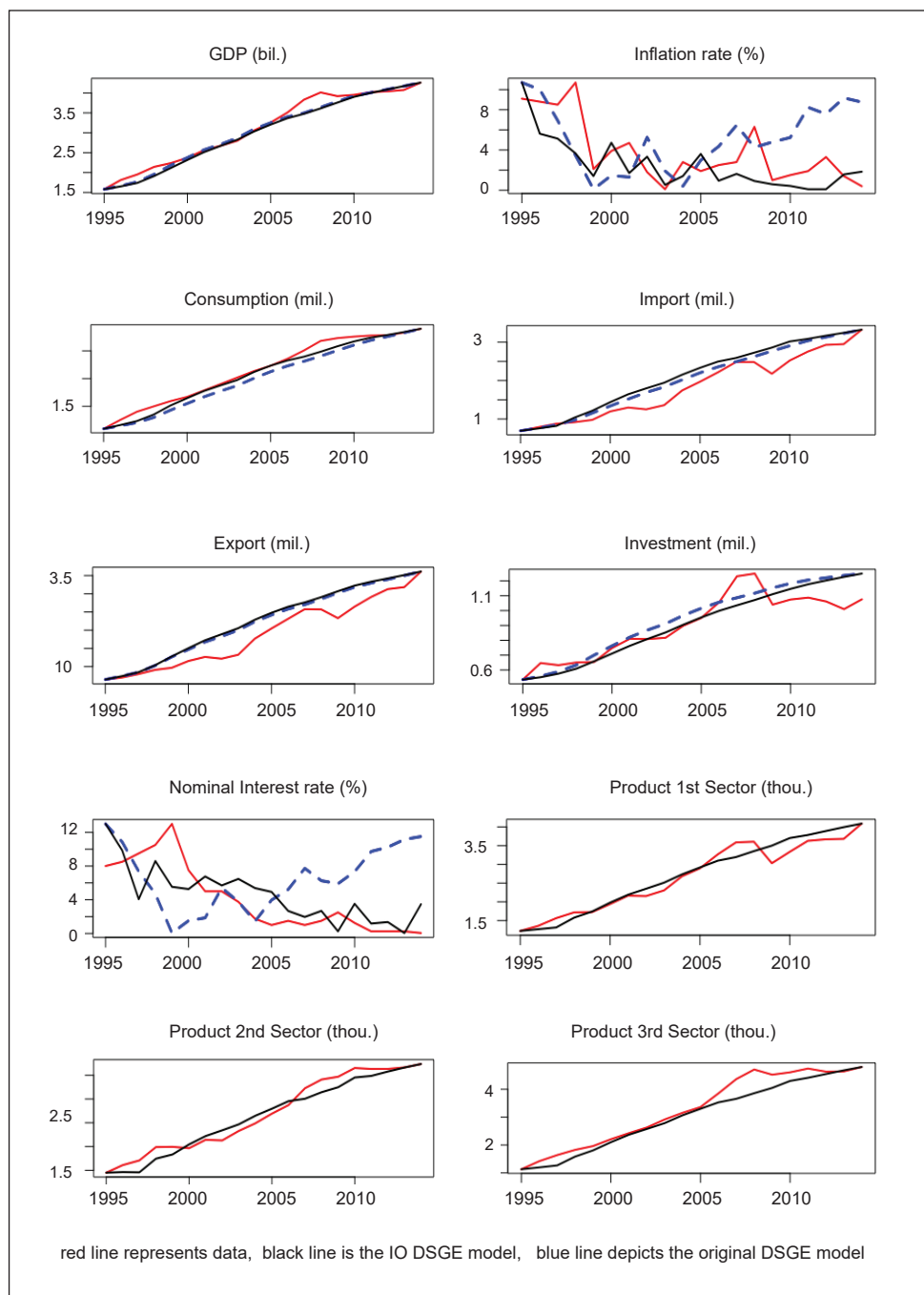
Source: Author's calculation.

Figure 2 | Monetary policy shock



Source: Author's calculation.

Figure 3 | Evolution of the Selected Variables over Time



Source: Author's calculations.

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