A MODEL OF GERMAN SPOT POWER MARKET

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Abstract:

This paper aims to model the day-ahead prices on the German EPEX SPOT exchange during the year 2011 using a fundamental mixed-integer programming model with focus on the changes in the volatility of prices. A model of the German market is built from publicly available data. Various constraints on the supply side such as operational characteristics of power plants are described, characterized and ultimately formulated as constraints of a cost-minimization problem. Unknown power plant characteristics are estimated by expert opinions or are inferred indirectly from other data. Several scenarios testing the impact of constraints and modelling approaches are analysed. In addition, a future scenario simulating the year 2016 is used to forecast price developments under the ongoing massive renewable energy growth. Finally, results are discussed with respect to price forecasting accuracy with a focus on the changes in the volatility of prices.

Keywords: power market modelling, mixed-integer linear programming, start-up costs
JEL Classification: Q41, C61

1. Introduction

The goal of this paper is to create and calibrate a fundamental market model for German spot prices using publicly available information. By combining information about the aggregate availability of power plants together with prices of fuels and emissions as well as estimated efficiencies, a full supply curve for each day is created, which is then intercepted with the appropriate hourly demand to derive hourly prices. This simple "merit order" model is further improved by including various inter-temporal constraints, such as ramping rates and spinning reserve requirements, as well as start-up costs turning the initially simple linear programming (LP) model into a mixed-integer linear programming (MILP) one. Simple calibration is performed by comparing market prices with the outputs of different scenarios, which are meant to test the impact of uncertain inputs or unknown constraints.

The obtained model has multiple uses ranging from simple what-if questions designed to understand the present market conditions – *e.g.* what would the market price be if the availability of lignite was lower – to more complex analysis. In this paper one such complex scenario is explored to provide answers about expected prices and their profiles under increasing contribution from renewable sources. In addition, the model could be put to daily use as a price forecasting tool, if a suitable day-ahead load forecast is provided.

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The structure of this paper is as follows. In Section 2 the various attempts to model German power prices from the literature are presented. In Section 3 the power market in Germany is described and the concepts of baseload and peak prices are defined as the simplest measures of daily volatility of demand. Additionally, publicly available data for the German power market are stated together with their transformation into a form suitable for the model. This transformation is the main advance over the existing research. In Section 4 the modelling of power markets by linear and mixed integer programming is described and the implications of various modelling assumptions are discussed. Section 5 contains a case study for the 2011 spot prices. Several model results are presented and discussed. In Section 6 the results are summarised and possible future research is outlined.

2. Literature Review

The literature on modelling of electricity prices is extensive and a comprehensive survey is beyond the scope of this paper. The main approaches are therefore described only in order to relate this work to the existing literature. Models for electricity prices are either based on fundamental approaches, directly describing supply and demand, or focus on modelling distributional properties of energy prices. The former approach essentially aims to explain the price as the cost of supplying the appropriate demand. Consequently, only the factors contributing to the cost of supply, such as the variable costs of the marginal power plant, are considered. In the most extreme, the latter approach can be used to model prices on its own, without any additional inputs required – a pure "time series" approach.

Most of the models fall somewhere in between those two theoretical extremes as both approaches have their merits. For example, a fundamental model may benefit from statistical approaches when unknown or inaccurate parameters are estimated from historical or residual data to improve the model fit.

The appropriateness of a model is also determined by the relevant time scale as the volatility of drivers also changes in time. Prices in the short term will be much more dependent on physical fundamentals, such as the level of demand, plant availability or generation from non-dispatchable sources. Conversely, long-term futures markets tend to behave like share prices as they are driven by more random inputs, such as volatilities of fuel prices. Similarly to other commodity markets, the development on the short-term market can influence the long-term markets and *vice versa*. Cointegration analysis can capture both the short and long-term relations among variables, *e.g.* Šumbera (2009).

The European Energy Exchange AG (EEX) is the most important energy exchange in continental Europe providing a spot market for Germany and several neighbouring countries as well as derivatives markets for fuels and emission certificates. Due to its importance and regional influence, many attempts to model and predict prices on EEX have been made. The literature review will focus on fundamental models only. Such models tend to be based on a LP/MILP framework as it allows to capture both variable and start-up costs. These models are frequently used to investigate the possible exercise of market power. Lang (2006) investigates the question of market power by using a successive MILP model followed by an ordinary LP model in order to overcome the fact that start-up costs are not reflected in shadow prices of the MILP. Müsgens (2006) also examines market power in Germany, but uses a multi-regional dispatch model consisting of Germany and 7 other countries. Due to the high number of plants in the model, the

individual units are aggregated into eight different generation technologies. Even though the model is solved using LP, it takes into account start-up costs *via* a linear simplification.

Fundamental models also allow to test the impact of major policy changes, such as German nuclear phase-out. Bruninx (2012) presents a detailed multi-nodal MILP model taking into account start-up costs, which represents Germany and the surrounding countries by means of 26 nodes. To test the impact of the phase-out, the author analyses 162 scenarios accounting for different load, renewable generation and cross-border interchange.

3. German Power Market

3.1 Market design and products

The German power market is one of the largest power markets in Europe due to the country's demand and its central location in Europe. Power with delivery to German grid can be traded both on and off exchange. Power market for delivery the next day – also known as the day-ahead or spot market – is mainly traded on the exchanges. The spot market for electricity is operated by EPEX SPOT, a joint venture owned by German EEX AG and the French Powernext SA. EPEX SPOT is the last liquid exchange for the zone where traders can offload their speculative positions[1].

In addition to hourly prices, two indices are computed each day: Phelix (Physical Electricity Index) Day Base and Phelix Day Peak. The former is the average of the 24 hourly prices for a day, while the latter is the average of hours 9 to 20 during working days of the week. These indices are used to calculate the settlement of futures contracts. The complement to the Peak index is the Off-peak index.

A useful measure of volatility of prices within the day is the ratio of peak to baseload prices, the peak-base ratio**.** The peak period, due to its coincidence with working hours and other times of economic activity, generally corresponds to the period with the highest demand. Consequently, prices in the peak tend to be 20%–40% higher than the baseload price, *i.e.* the typical peak-base ratio is in the 1.2–1.4 range.

3.2 EEX Transparency power market data

Market transparency in continental Europe is quite limited, in spite of recent EU regulations. In Germany, the situation is additionally aggravated by the existence of four control zones managed by four different TSO companies. In addition to several vertically integrated players, there are approximately 850 municipal energy distributors (Stadtwerke) in Germany, many of which have their own generators. Integrating information from such a high number of sources is problematic and, consequently, it is difficult to find detailed data covering whole Germany. For example, while there are several demand statistics available in hourly resolution, their sums do not match the official figures from German statistical office available in monthly detail. Similar difficulties arise on the supply side as well. A comprehensive power plant list published by the German Environment Ministry, Umweltbundesamt (2012), lists name, fuel, technology, size and age of German power

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1   The later, intraday market has much lower liquidity than the spot market. In 2011 total traded
     volume on the spot market was 224 TWh compared to only 14 TWh on the intraday market.

plants bigger than 100 megawatt (MW). However, basic technical information such as efficiency which is essential in order to calculate plant's variable costs is not reported.

The EEX exchange publishes market data on its website in order to increase the market transparency and hence the liquidity. Historically, the publication of transparency data was voluntary but in 2009 additional, statutory transparency data started to be published, EEX (2012). The voluntary data includes daily published information such as (forward looking) installed capacity, available capacity, planned and unplanned outages. Historical generation on hourly resolution is also available.

However, the data is anonymised *via* aggregation, hence only the totals for each fuel-type (nuclear, lignite, coal, gas, oil, hydro) are available. In addition, the data can suffer from perimeter changes depending on individual company's decision to start or stop reporting this voluntary data. To overcome these shortfalls as well as to conform to EU requirements, the statutory transparency data is published. In spite of covering the same type of information, the statutory data is defined on a different but still incomplete perimeter (all power plants with installed capacity over 100 MW) and is anonymised again. Nevertheless, the statutory data covers some new areas such as actual and forecasted wind and solar generation.

3.3 Data transformation

Given the above complexities with data availability, several simplifying assumptions and transformations were made:

– Every power plant in Germany was characterised as either dispatchable, non-dispatchable or excluded.

– Dispatchable plants were modelled economically in the highest detail possible, non-dispatchable plants were aggregated by plant fuel–type and modelled as must-runs[2] and excluded plants were omitted from the model.

– Since not all German power plants were included in the model, the traditional load measure (country-wide load) could not be used. Instead, total generation from all the plants included in the model (dispatchables + non-dispatchables) was used as the load.

These assumptions and transformations are graphically summarised in Figure 1 below and explained in detail in the following paragraphs.

Data sources. Three data sources were used: EEX (2012) statutory and voluntary data and Environment Ministry power plant list, Umweltbundesamt (2012). Voluntary dataset was used as the base set since statutory data does not provide hourly generation by fuel-type (except for wind and solar). In addition, the voluntary dataset has a good representativeness of the conventional plant park as shown in Table 1.

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2   A power plant, whose generation is determined by its availability only, modelled as having zero variable costs.

Table 1 | Comparison of Maximum Net Generating Capacity in Germany (MW)

Conventional power plants	Nuclear	Lignite	Coal	Natural Gas	Oil	Derived gas
EURELECTRIC	20,480	20,348	27,929	23,353	6,116	4,610
EEX Voluntary data	20,480	19,946	17,136	9,024	1,506	N/A
Case study	20,480	19,946	17,136	9,024	N/A	N/A
Representativeness	100%	98%	61%	39%	0%	0%

Source: EURELECTRIC (2011), EEX (2012), own calculation

Dispatchable plants. Lignite, coal and gas power plants from the voluntary data were matched against the Environment Ministry list in order to determine their age, Combined Heat and Power (CHP) status (Y/N) and in the case of gas-fired plants also their technology (CCGT, Gas ST, GT[3]). Technical and economic characteristics such as plant efficiency, minimum stable generation or start-up costs were defaulted using expert estimates for a given technology and age group. The defaulting values are listed in Table 2 and are in reasonable agreement with the assumptions found in Bruninx (2012).

**Table 2 | Power Plant Characteristics for Dispatchable Plant Types**

Plant Characteristics / Plant Type	Lignite	Coal	CCGT	Gas ST	GT
Efficiency [%] **(based on commissioning date):**					
Before 1970	31%	33%	38%	34%	24%
1970–1979	32%	34%	39%	35%	25%
1980–1989	35%	36%	43%	39%	30%
1990–1995	35%	36%	48%	39%	30%
1996–2005	38%	39%	53%	43%	35%
2006–present	42%	43%	57%	43%	35%
Efficiency decrement for plants smaller than 300 MW [%]	-2%	-2%	-3%	-3%	-3%
Variable O&M Cost [EUR/MWh]	3	2.8	2.7	0.8	2
Unit Start-up Costs [EUR/start-up/MW]	180	100	25	40	20
Minimum stable generation [% of Max Capacity]	60%	60%	50%	40%	25%

Source: Own calculation

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3   Combined-Cycle Gas Turbine, Gas Steam Turbine, Gas Turbine.

Non-dispatchable plants. Nuclear generators were aggregated into a single non-dispatchable power plant and treated as a must-run. Solar and wind fuel-types were also treated as must-run and their availability was set equal to their historical generation from the statutory data.

Figure 1 | Treatment of Various Data Sources in the Model
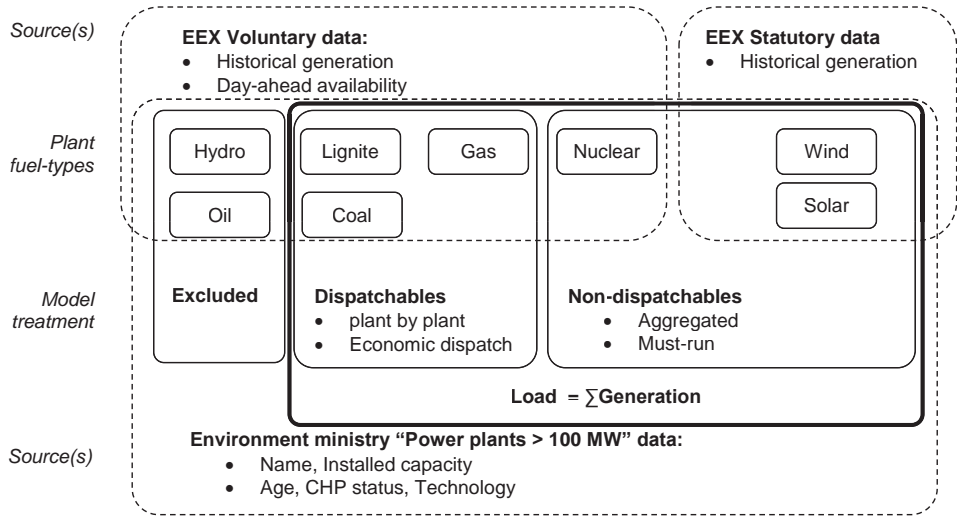
Source: Own calculation

Power plants excluded from the model. Data regarding power plants of the oil and hydro fuel-types was removed from the set. The installed capacity of oil-fired generation is only 6 GW and its historical generation series has many and frequent gaps. Apart from pump storage, most of hydro capacity in Germany is run-of-river[4], *i.e.* a must-run, with volatility profile much smaller than that of solar or wind. In addition, power plants not included in the voluntary dataset were (automatically) excluded from the model.

Commodity prices. The EEX exchange has liquid trading in both coal and gas products as well as emission allowances of the EU ETS. The daily closing prices for month-ahead (coal) and day-ahead (gas, CO2) products were taken as the relevant fuel prices (see Figure 2). An estimate of transportation cost was made for coal, since the usual quotation price is for delivery in ARA[5], not in Germany. Conversely, lignite, due to its low energy density and consequent high transportation cost, is a local fuel. An expert estimate of 1.2 EUR/GJ was used for lignite price (including transportation cost). This figure falls between the low case of 1.15 EUR/GJ and the reference case of 1.43 EUR/GJ found in Schneider (1998). Other defaulting values for fuels are listed in Table 3.

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4   See: EURELECTRIC (2011). Modelling pump storage stations is beyond the scope of this paper.

5   Amsterdam, Rotterdam, Antwerp.

Table 3 | Commodity Properties

Fuel Characteristics/Fuel	Lignite	Coal	Gas	CO²
Emission factor [tCO2/GJ]	0.111	0.0945	0.0561	N/A
Fuel cost [EUR/GJ]	1.2	Daily (from EEX)	Daily (from EEX)	Daily (from EEX)
Transportation cost [EUR/GJ]	0	0.3	0	0

Source: Own calculation

Load. Load was defined as the sum of generation of the non-dispatchable (solar, wind, nuclear) and dispatchable (lignite, coal, gas) plants. Even though the analysis could be carried out without the non-dispatchables – as they contribute the equal amount to both demand and supply – they were included to improve the representativeness of the final model. Not only is their contribution significant to the total supply, moreover the specific hourly profile of renewables means that by including them the load profile used in the model is much closer to the profile of the total German load. In addition, by including the non-dispatchables in the model, it is easy to test the sensitivities to their volume of generation.

Figure 2 | Commodity Prices (2011) Used : Lignite, Coal, Gas (in EUR/GJ), CO² (in EUR/t)
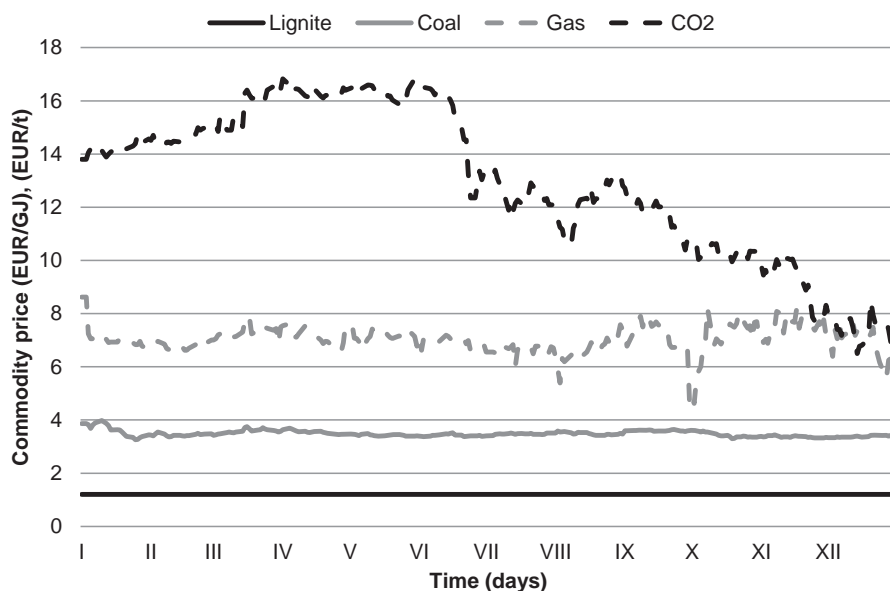

Source: Own calculation for lignite, EEX (2011) for other commodities

The resulting dataset has the following properties:

- There are three dispatchable plant fuel-type groups modelled endogenously: lignite, coal and gas. Each group has known availability (daily) and generation (hourly) profiles as well as known or estimated economic and operational characteristics.
- There are three non-dispatchable fuel-type groups modelled exogenously: nuclear, wind, solar. Each group has a pre-determined generation (hourly) profile.
- The total load to be supplied from these six groups is known.

Figure 3 shows the resultant supply curve, *i.e.* the merit order of the dataset assuming average availability together with an average level of demand.

**Figure 3 | 2011 Supply Stack of German Power Market Used in the Study**
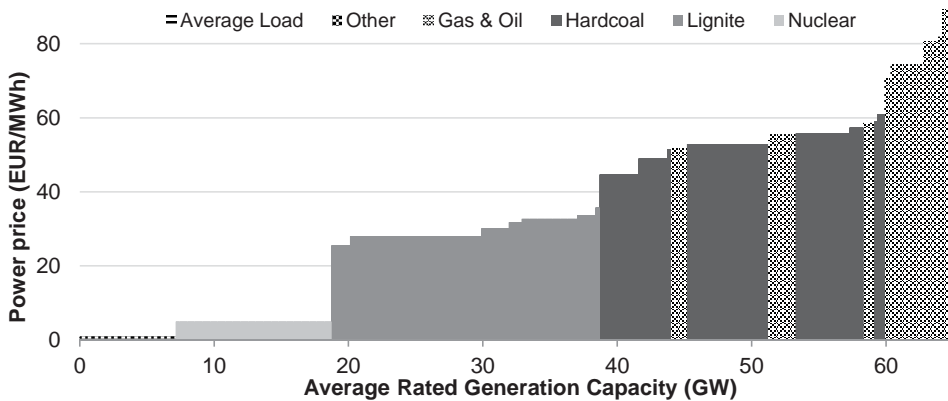
Source: Own calculation

3.4 Representativeness

As electricity cannot be stored, total demand (load) has to equal the total supply (generation) at every instant. The same is true for any subset of the total. Moreover, as long as the subset covers the total set homogenously, *i.e.* the subset includes samples for each region of the stack, the prices obtained on the subset will be sufficiently close to those obtained on the total set. If the subset creates no big gaps in the merit order, the reduced load will intersect the reduced supply curve on a point whose variable costs are either the same as in the total set (the original marginal plant is in the subset) or whose variable costs are close enough (it is not).

Figure 4 | Price Similarity on a Representative Subset, Prices Obtained on a) the Full Set, and b) the Reduced Subset are Similar as Long as the Reduced Subset Is Representative Enough
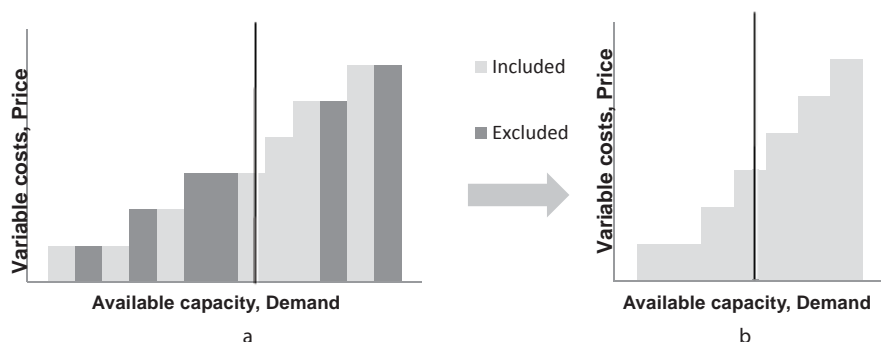
Source: Own calculation

The dataset used in the case study has been compared with the latest European statistics from EURELECTRIC (2011), which lists the net installed capacity for conventional power plants in Germany in 2009. In spite of some error introduced by comparing different years, the representativeness of the used dataset is fairly high for both lignite and coal, and reasonable for gas plants (see Table 1). On the other hand, the efficiencies of gas plants in the used dataset range from 32% to 50% spanning the full permissible range and hence creating no bias. Renewable representativeness is 100% by definition.

4. Modelling Framework

4.1 Nomenclature

Indices

i	generators, $i \in 1..I$
t	periods in the modelling horizon, $t \in 1..T$
f	fuel-types, $f \in$ {lignite, coal, gas}
M	months, $M \in$ {January, …, December}

Decision variables

p_{it}	Production quantity in period t [MW]
x_{it}	Unit commitment variable in period t (1 if online, 0 otherwise)

Objective function terms

c^p_{it}	Production cost in period t [EUR]
c^u_{it}	Start-up cost of generator i in period t [EUR]

Parameters

P^{min}_i, P^{max}_i	Minimum stable generation and Maximum generation capacity [MW]
B_{it}	Linear coefficient of a linear production cost curve (constant efficiency)
FO^u_{it}	Fuel off-take at start-up in period t [GJ]
FP_{it}	Fuel price in period t [EUR/GJ]
EP_{it}	Emission price in period t [EUR/ton of emissions]

EMF_i	Emission factor [tons of emissions/GJ]
E_i	Efficiency
V_i	Other variable costs [EUR/MWh]
C^u_i	Start-up cost [EUR]
RR^u_i, RR^d_i	Ramp rate up and down [MW/h]
Dem_t	System demand in period t [MW]
Res^+, Res^-	Positive and negative reserve requirements [MW]

4.2 Basic formulation

In time t, power plant i generates p_{it} units of energy:

$$p_{it} \geq 0, \forall t \in T, \forall i \in I \tag{1}$$

The total generation equals demand

$$\sum_i p_{it} = Dem_t, \forall t, \tag{2}$$

subject to maximum capacity constraints

$$p_{it} \leq P_i^{max}, \forall t, \forall i, \tag{3}$$

and other operational constraints, which can, including (2), be jointly expressed in matrix notation as

$$Ap_{it} \leq b. \tag{4}$$

The goal is to minimise the total costs of generation of all generators in all periods and can be expressed as an objective function

$$\min \sum c_{it} \, p_{it}. \tag{5}$$

Then Equations (1), (4) and (5) define a LP problem, which can be solved using standard methods.

Next there are the most important aspects of models based on the above framework:

− If cardinality of T is 1, *i.e.* the **model horizon length** is 1 (hours are solved one by one), then this approach is a simple "merit order" model. Conversely, including **inter-temporal constraints** requires a horizon length longer then 1. Such constraints usually smooth prices as they typically constrain the total generation of a power plant which will consequently run preferably in periods with the highest prices.

− The modelling of **start-up costs** requires binary decision variables to model the offline/online state of each power plant together with the necessary inter-temporal constraints resulting in a MILP problem.

− The **system price** is the cost of supplying an additional unit of energy, which is the definition of the shadow price of the demand constraint (2). This price will be used as the predictor of the market price.

4.3 Constraints

Each generator can produce electricity within a certain range of values. The upper and lower limits – the **maximum capacity** P^{max} and the **minimum stable generation** P^{min} – typically refer to the limits of sustained generation under normal operating condition. These **operating limits** can be expressed as constraints

$$p_{it} \leq x_{it} P_i^{max}, \forall t, \forall i \ , \tag{6}$$

$$x_{it} P_i^{min} \leq p_{it}, \forall t, \forall i , \tag{7}$$

where a binary variable x_{it} is required to capture the unit's (offline/online).
To capture the information about **day-ahead availability** from voluntary transparency data, constraints limiting the maximum production from all plants with the same fuel-type during a given day are formulated as

$$\sum_{i \in F} p_{it} \leq P_{f,D}^{max}, \forall t, f \in \{lignite, coal, gas\} \tag{8}$$

Due to the high number of plants in the model, preference has been given to simplicity of formulation over a detailed, but more accurate one. Consequently, the **variable production costs** are formulated using a linear relationship (even though in reality they are typically approximated using a 2nd or 3rd degree polynomial)

$$c_{it}^p = B_{it} p_{it}, \forall t, \forall i , \tag{9}$$

where B_{it} is a term corresponding to the total unit variable costs, *i.e.* the sum of variable fuel, emission and operation & maintenance costs *per* unit of output

$$B_{it} = (FP_{it} + EMF_i EP_t) 3.6 / E_i + V_i, \forall t, \forall i . \tag{10}$$

Similarly the formulation of **start-up costs** has also been simplified. While in reality these costs typically depend exponentially on the number of hours since the last shutdown, Christober (2011), in this paper a single start-up cost is assumed

$$c_{it}^u \geq C_i^u (x_{it} - x_{i,t-1}), \forall t, \forall i. \tag{11}$$

Thermal units have limits on ramping up and down their production (in order to avoid thermal stresses leading to failure or increased maintenance costs)

$$p_{it} - p_{i,t-1} \leq RR_i^u, \forall t, \forall i , \tag{12}$$

$$p_{i,t-1} - p_{it} \leq RR_i^d, \forall t, \forall i . \tag{13}$$

This formulation works for units without a minimum stable generation P^{min} and also if the ramp rates are bigger or equal to the minimum stable generation ($P^{min} \leq RR^u$ and $P^{min} \leq RR^d$). In the opposite case, the equations need to be modified to allow for the unit to start-up, *e.g.*

$$p_t - p_{i,t-1} \leq (RR_i^u + P^{min}) x_{it} - P^{min} x_{i,t-1}, \forall t, \forall i . \tag{14}$$

In addition demand-supply balance, captured in (2), generators which are online must be able to handle unforeseen changes in demand, which are expressed as positive and negative **reserve requirements**

$$\sum_i x_{it} P_i^{min} \leq Dem_t - Res^-, \forall t \,,$$ (15)

$$\sum_i x_{it} P_i^{max} \geq Dem_t + Res^+, \forall t \,.$$ (16)

Finally, the objective function to be minimised is the total cost of generation expressed as sum of production and start-up costs, replacing the simple formulation (5)

$$min \sum_t \left(c_{it}^p + c_{it}^u x_{it} \right).$$ (17)

To capture the information from historical voluntary transparency data, constraints representing **must-run generation** of gas CHP units are formulated similarly to the available capacity constraints above. On the other hand, these constraints are formulated with a different right hand side for each month

$$\sum_{i \in F} p_{it} \geq P_{f,M}^{min}, \forall t \in M, f \in \{gas\} \cap \{CHP\}.$$ (18)

The chart below shows the minimum generation from gas-fired plants for each month of 2011. A baseload component of approximately 600 MW together with a seasonal profile driven by the temperatures of 0–700 MW (heating) are clearly visible. These values were used as the RHS in the constraints (18). The shape of the seasonal profile is consistent with CHP generation profile for all German power plants as reported by Müsgens (2006).

Figure 5 | Seasonality of Gas-Fired CHP Plants : Minimum Hourly Generation from Gas-Fired Plants for Each Month of 2011 (Assumed CHP Constraint) [MW]
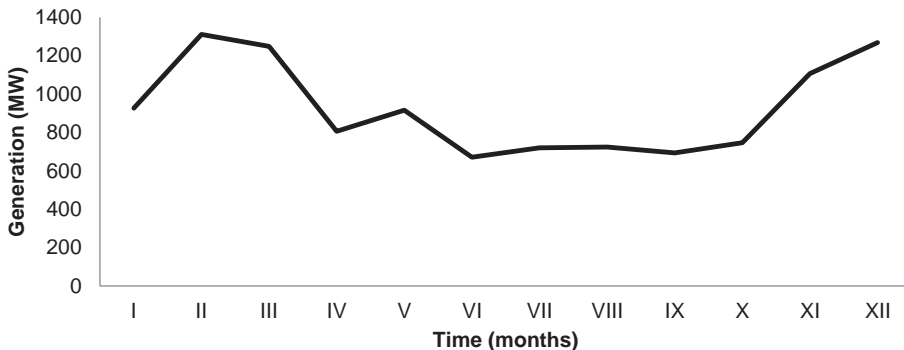
Source: Own calculation based on EEX (2012)

5. Case Study

The case study aims to model the prices on the EPEX SPOT exchange for the year 2011, *i.e.* to model 24 values for each day of the year as determined by the exchange on a day-ahead basis. The models are populated with numbers obtained from public data

sources complemented with authors' own expert assessment of some power plant characteristics. To test assumptions about the modelling logic as well as to characterise uncertainty in key drivers, several modelling scenarios are evaluated using metrics chosen to accurately capture spread between the Peak and Off-peak prices. Additional scenarios simulate the upcoming changes to German energy market such as nuclear decommissioning and massive growth in renewable energy sources.

5.1 Scenarios

This data was used within a power market modelling software *PLEXOS for Power Systems*[6]. A base case model (BC) for the year 2011 was set-up, which includes all of the above constraints except for reserve requirements, ramp rates and run-up rates.

Additionally, six sensitivity models were set up. Two sensitivities are designed to show the uncertainty arising from expert estimates:

– Base model with efficiencies decreased by 1 percentage point (E-1pct).
– Base model with gas price increased by 10% (G1.1).

Two other sensitivities illustrate the addition of some constraints:

– Base model with positive and negative reserve requirements set to 2 GW (R2GW).
– Base model with run-up rates and ramp rates constraints included (Ramps).

Finally, two modelling sensitivities discard the integrality assumptions:

– Linear relaxation of the Base model (BC-LR).
– Linear relaxation of the Base model without start-up costs (Merit order).

In addition to the above seven models for the year 2011, a hypothetical future development scenario intended to simulate the year 2016 was set up. Only few selected drivers were changed from the base case.

The **nuclear** profile was changed from the historical profile to piecewise-constant generation profile derived from future installed capacity of 12.1 GW and historical monthly load factors (intended to capture seasonality of outages). **Renewable profiles** were scaled to match the assumed trajectory to 2020 targets. Wind was scaled with the factor of 1.4 and solar with a factor of 2[7]. Apart from this scaling, the renewable profiles were not changed (essentially assuming the same meteorological conditions in 2016 as in 2011). **Load** was not changed. The rest of the **power plant portfolio** was kept the same including its variable costs. This is a simplification as in reality between 2011 and 2016 new plants will be built and old plants will be decommissioned and commodity prices will likely change. On the other hand, this assumption makes the whole analysis more "*ceteris paribus*".

Power prices in the model were artificially capped between -5 and 120 EUR/MWh. The above-mentioned combination of unchanged load and increased must-run generation leads situations where must-run generation in the model is higher than load. In practice, this would be resolved by curtailing some of the wind and solar generators. At the same time, the prices on the exchange would turn negative. To simulate this, a virtual demand

6   PLEXOS® Desktop Edition [programme] by Energy Exemplar Pty Ltd.

7   This corresponds to 98 TWh/y which is consistent with the 2016 NREAP (2010) target of 105 TWh/y.

node capable of absorbing excess generation at a price of -5 EUR/MWh was added to the model. The total volume of absorbed energy is approximately 1% of the total generation. Approximately 220 hours are impacted, most of them occurring between April and October, *i.e.* due to excessive solar generation. The total impact on baseload price due to these hours is approximately -1.3 EUR/MWh. The inverse situation – load exceeding supply – occurs during 24 hours in Q1 due to decreased nuclear generation. The prices in those hours were capped at 120 EUR/MWh.

Two scenarios for the year 2016 were created:

- Future case (FC) like the 2011 base case but with the above adjustments.
- Linear relaxation of the future case (FC-LR).

All models use a 24-hour horizon, *i.e.* solving one year means solving 365 steps of 24 hours each. This is consistent with the daily price-formation mechanism.

Figure 6 | German Nuclear Availability Profiles: 2011 – Historic, 2016 – Assumed
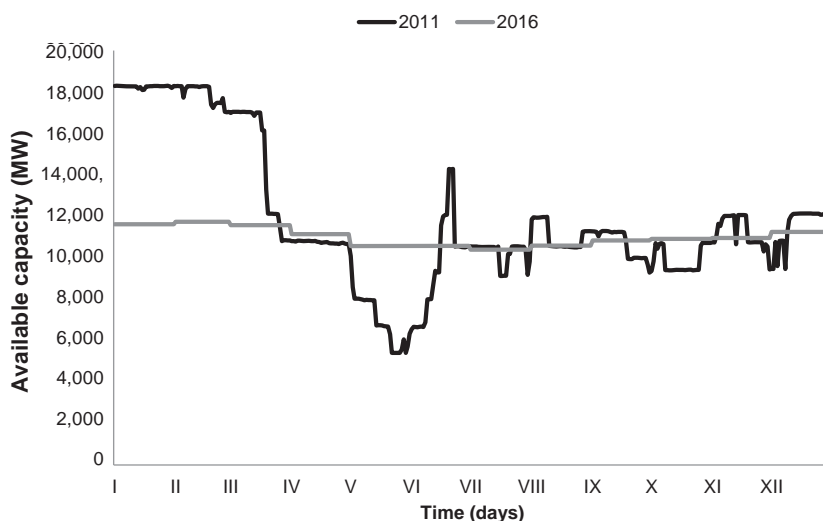
Source: 2011 – EEX (2012), 2016 – own calculation based on ENTSOE (2012)

5.2 Metrics

In order to compare the quality of the models and assess the development of prices, several different metrics are used.

Average level fit: Annual average prices for baseload (BL), peak (P) and off-peak (O) periods as well as the peak-base ratio. This group of metrics allows to determine how well a scenario matches annual average price as well as prices in periods of low and high demand.

Hourly level fit: the Mean Absolute Error (MAE) for all hours and peak and off-peak subsets. The second group of metrics is similar to the first one but takes into account the quality of fit hour by hour.

Measures of volatility: the average absolute deviation from annual and daily means of a series. The third group characterises the volatility of the 8,760 values on their own, *i.e.* without a reference to the historical outcomes. Consequently, its main use is to compare models representing history with those representing future states.

5.3 Results

Model results are summarised in the tables below. Table 4 compares general model characteristics such as the number of linear and binary variables, the problem size and computation time. Average and hourly level fits (in EUR/MWh) are shown in Tables 5 and 6, while Table 7 contains volatility measures in EUR/MWh.

Table 4 | Case Study Results: Comparison of General Model Characteristics * = Same as BC

	EEX	BC	E-1pct	G1.1	R2GW	Ramps	BC-LR	Merit order	FC	FC-LR
NNZs	N/A	36,634	*	*	86,410	75,107	*	3,192	*	*
Variables	N/A	8,712	*	*	19,848	14,370	*	6,504	*	*
Binaries	N/A	2,760	*	*	*	3,138	0	0	*	0
Comp. time (s)	N/A	285	369	334	1,718	4,285	137	42	352	100

Source: EEX (2011), own calculation

Table 5 | Case Study Results: Comparison of Average Levels (EUR/MWh)

	EEX	BC	E-1pct	G1.1	R2GW	Ramps	BC-LR	Merit order	FC	FC-LR
VALUE-BL	51.1	49.3	50.5	50.1	48.1	35.0	51.5	49.9	45.7	47.9
VALUE-P	61.1	55.8	57.1	57.6	55.4	50.8	59.7	53.8	49.0	51.2
VALUE-O	45.6	45.7	46.9	46.0	44.0	26.3	46.9	47.8	43.9	46.0
VALUE-P/B	1.20	1.13	1.13	1.15	1.15	1.45	1.16	1.08	1.07	1.07

Source: EEX (2011), own calculation

Table 6 | Case Study Results: Comparison of Hourly Level Fits (EUR/MWh) (the Values for Various Scenarios Are Shown as Differences to EEX)

	EEX	BC	E-1pct	G1.1	R2GW	Ramps	BC-LR	Merit order	FC	FC-LR
MAE-BL	0	6.5	6.4	6.3	6.8	16.9	7.0	7.4	12.3	13.4
MAE-P	0	7.1	6.5	6.0	7.3	11.1	7.9	8.3	15.9	16.6
MAE-O	0	6.2	6.3	6.5	6.5	20.1	6.6	6.9	10.3	11.6

Source: EEX (2011), own calculation

Table 7 | Comparison of Volatility Measures (EUR/MWh)

Deviation from	EEX	BC	E-1pct	G1.1	R2GW	Ramps	BC-LR	Merit order	FC	FC-LR
annual average	10.3	8.0	8.1	9.1	8.9	18.4	9.3	5.4	13.3	14.9
daily average	8.6	6.1	6.2	7.0	7.0	10.7	7.7	2.7	10.2	12.2

Source: EEX (2011), own calculation

Figure 7 | Model Results: Hourly Prices for the Week 20-26/6/2011 : EEX – Historical Prices, BC – 2011 Base Case, FC – 2016 Future Case (Strong Renewable)
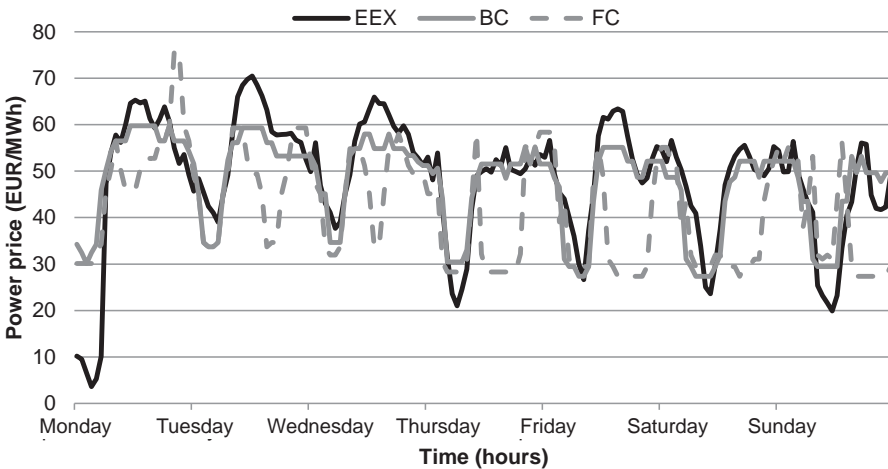
Source: EEX (2011), own calculation

The following conclusions can be drawn about the historic models (2011):

– All models (except for Ramps) show reasonable values for the absolute level of prices, in baseload, peak and off-peak.

– Average model deviation from true prices is about 6–7 EUR/MWh.

– All models (except for Ramps) underestimate the peak-base ratio as well as the deviations from both annual and daily means.

– Uncertainty in heat rates (E-1pct) impacts prices by approximately 1 EUR/MWh.

– Models could be fine-tuned for each fuel-type basis to improve the fit. As the results of the model with 10% higher gas prices (G1.1) show, this model has better average and hourly level fits as well as better volatility measures than the base model.

– The addition of reserve requirements (R2GW) decreases prices and increases computation time by an order of magnitude.

– Simple linear relaxation of the base case model (BC-LR) achieves a reasonable fit, fast computation time and the best Peak fit.

– Merit order approach does not capture the daily volatility of prices especially in the off-peak.

The two future models show that:

– The same structural relationship between a MILP model and its linear relaxation holds (somewhat higher prices but more than twice the computation speed for the relaxed model).
– Peak-base ratio will decrease in 2016, while the average deviations from both daily and annual prices are set to increase significantly.
– As Figure 7 shows, the hourly profile will be altered due to increased penetration of renewable energy source especially in the March–October period when typical daily minimum will be reached during peak hours.

6. Conclusion

To summarise, this paper presents a fundamental spot market model which uses publicly available information such as day-ahead power plant availabilities. Even though it works only with a subset of all generators on the market, the model can create an hourly price forecast. If the power plant subset has reasonable representativeness of the total set, the model will capture the daily prices, although it still underestimates the peak-base ratio. Several scenarios are presented which capture uncertain inputs as well as illustrate the impact of different modelling assumptions. While the scenarios which model start-up costs explicitly still underestimate the peak-base ratio, they are better than a simple merit order approach.

Some of the notable results of these scenarios are:

– Uncertainty in estimated efficiencies is approximately 1 EUR/MWh *per* efficiency percentage point.
– Inclusion of ramp rates in the model improves the hourly characteristics such as the peak-base ratio or the deviations from annual and daily means.
– Addition of reserve requirements decreases prices and increases computation time by an order of magnitude.
– Linear relaxation of the originally integer problem can achieve a reasonable fit with a fast computation time as long as start costs are taken into account (*i.e.* simply using a merit order approach is not sufficient to capture the volatility of prices).
– Peak-base ratio will decrease in 2016, while the average deviations from both daily and annual prices are set to increase significantly. At the same time the hourly profile will be altered due to increased penetration of renewables especially in the March–October period when typical daily minimum will be reached during peak hours.
– As the hypothetical 2016 scenario shows, in case that the energy generated from volatile renewable sources such as wind and solar continues to increase, the volatility as measured deviation from daily average will increase causing more frequent start-ups of conventional time. At the same time the peak-base ratio will decline, indicating that this standard measure is no longer the appropriate one.

6.1 Further calibration

The model could be additionally fine-tuned on a fuel-type basis. For example, as the total demand of this system was designed as the sum of generation profiles of six fuel-types, one could further calibrate the model to match the given generation profile of each fuel-type. Correctly capturing the generation by fuel-type, especially of the marginal fuel-type should improve the price forecast. On the other hand, the actual generation happens 12–36 hours after the day-ahead prices are formed. In the meantime, a power plant may unexpectedly come offline, possibly shifting the stack and consequently the generation profile. This shift is not captured in the day-ahead information about availability and hence cannot be replicated by the model.

Moreover, calibrating to generation by fuel-type can be problematic on its own. For example, for most of the fourth quarter the available lignite capacity was never fully utilised, even during hours when more expensive coal and gas-fired plants were dispatched. While this could be explained rationally (*e.g.* the plant was available, but was constrained by emission or fuel constraint), an incorrect reporting is equally likely. The generation from the model led to full utilization of all available lignite capacity, thus leading to lower prices. In this case, the model would probably benefit from some form of adjustment.

Unfortunately, it is very difficult to distinguish an outlier from a lasting change. One way to handle these modelling difficulties is with a short-term auto-regressive process[8].

6.2 Model extensions

The model could be further improved using the following refinements and extensions:

– More detailed supply curve formulation by modelling varying efficiency for the thermal power plants or the start-up cost as a function of time offline. This refinement will result in a structurally similar, but more detailed model.

– Using generation forecasts instead of actual generation for the non-dispatchable fuel-types. This change will lead to a more consistent formulation, avoiding the problems arising when the day-ahead forecast is vastly different to the actual outcomes.

– Inclusion of other dispatchable fuel-types such as endogenous modelling of pump-storage and hydro reservoir power plants. This would require capturing various hydro-specific constraints whose details are not publicly reported. Conversely, such power plants can influence prices in spite of having no intrinsic cost of their own merely by having the ability to switch generation from one period to another.

– Model extension to use a country-wide load signal. The load used in the current formulation is defined as the sum of generation of the non-dispatchable and dispatchable plants. Consequently, there is no simple transformation between the used load and the traditional load measure, the country-wide load. In fact, the difference between the two load signals is mostly the generation from the power plants, which do not report to the EEX. While their generation may be relatively stable from one

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8 One could calculate the error between the forecasted and the actual generation and assume this error to persist into the next day.

day to the next, it will vary significantly throughout the year (as many of them are CHPs), making the finding of a suitable transformation very difficult.

– Model extension to include cross-border flows (imports and exports). The inclusion of flows is problematic due to several reasons. First, Germany has ten different interconnections with its neighbours. Due to the existence of transit flows, the net flow would have to be included as opposed to the incorrect inclusion of only some interconnectors. Correctly estimating the price of the net flow when it is a net import is made problematic by the high number of borders. Second, the extra information on net flow is provided when it is a net export only when the model is calibrated to use the country-wide load (see above) as only then it makes sense to sum the two (due to the identity generation = load + net exports). Finally, it becomes more difficult to use the model in a forecasting regime as the quantity of input data that has to be forecasted increases significantly.

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