THE UNCOVERED PARITY PROPERTIES OF THE CZECH KORUNA

Alexis DERVIZ*

Abstract:

The paper studies the compliance of the CZK – EUR exchange rate with the uncovered parity of returns on assets denominated in the two named currencies. A comparison with the same property for the euro-dollar rate is made. An uncovered total return parity (UTRP) formula is derived from the equilibrium in a portfolio optimization model with liquidity constraints. It is shown that the uncovered parity of total returns, and not of short-term money market rates, is a natural outcome of stochastic equilibrium asset pricing models that generalize the International Consumption-based Capital Asset Pricing Model. Accordingly, the traditional uncovered interest rate parity should be replaced by UTRP in empirical analysis. UTRP tests for the CZK/EUR and the USD/EMU currency pairs are conducted using yields of long-term government bond yields. UTRP typically holds, although the time horizons and measures of exchange rate movements, for which it becomes visible, may vary.

Keywords: uncovered parity, asset prices, portfolio optimization, international consumption-based capital asset pricing model

JEL Classification: F310, F410, E440, G110, G120, C610

1. Introduction

The present paper investigates the abidance of the CZK by the theoretically and empirically controversial rule of the uncovered parity of national interest rate levels. In doing so, it seeks to remove a long-lived misunderstanding, which erroneously associates the textbook uncovered interest rate parity (UIP) property of the expected exchange rate with differences in short term money market interest rates across countries. Empirical finance literature has by now almost completed deconstructing the said "money market" UIP. Particularly, in a transitive economy where applicability of most international macroeconomic shortcuts must be scrutinized anew, a mechanical implementation of the theoretically and econometrically flawed standard UIP concept is unable to provide useful insights. I am proposing a micro foundation

of the "right" exchange rate parity based on portfolio optimization under uncertainty in continuous time under convex transaction costs. I suggest a replacement of UIP by the total return parity of long-lived assets, to be used in both abstract reasoning and empirical work. The uncovered total return parity equation is applied to the CZK/DEM and /EUR exchange rates. For comparison, the same exercise is conducted for the USD/DEM and USD/EUR rates.

In its most general form, the model yields the Generalized Uncovered Total Return Parity (GUTRP) condition, related to the continuous time Consumption-based Capital Asset Pricing Model (CCAPM, see Duffie and Zame, 1989). I expose it to a test associated with the so-called Siegel's paradox (see Siegel, 1972). As it is well known, the latter discloses the inability of the uncovered interest rate parity to serve as an estimate for the future spot exchange rate due to the Jensen inequality effect. Siegel's observation about the asymmetry in the naive no-arbitrage argument leading to UIP indicates that there exists no genuine theoretical support for the presumed ability of interest rate differentials to predict future exchange rates.

Empirical evidence on the forward premia on practically every freely floating currency shows that the forward exchange rate is a severely biased estimate of the future spot exchange rate. Transitive economies, including the CR, are no exception (Figure 1 illustrates the latter case). The UIP hypothesis, a building block of almost every international macroeconomic model since Fleming and Mundell, was rejected in the studies covering the 1970s (Meese and Rogoff, 1983), the 1980s (Froot and Thaler, 1990), and the first half of the 1990s (Engel, 1996). Accordingly, since the covered interest rate parity (the equality of the international interest rate differentials and the corresponding forward exchange rates) is always satisfied for trivial deterministic arbitrage reasons, one arrives at the systematic failure of interest rate differentials to predict the spot rate correctly. McCallum (1994) summarizes this experience by noting that the sign of the coefficient in the regression of the
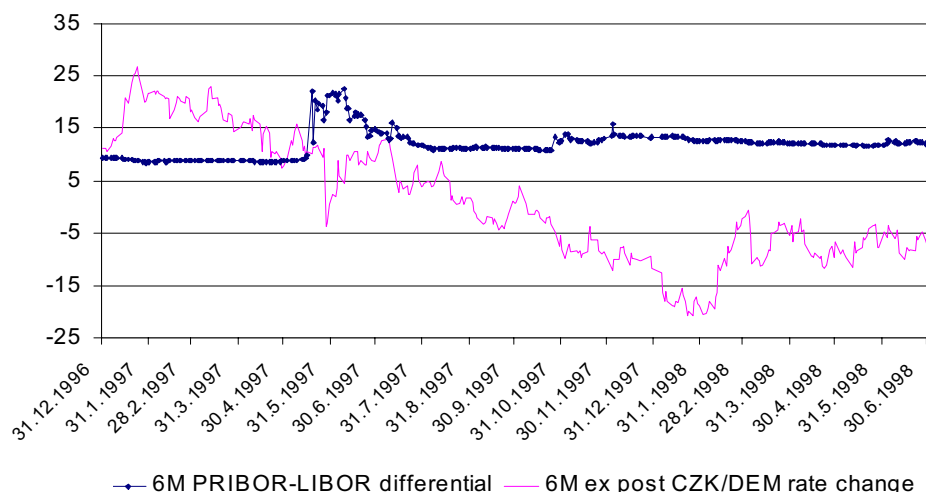
Figure 1
Czech and German Interbank Interest Rates and the CZK/DEM Exchange Rate

a) Three Month Interest Rates and Exchange Rate Changes

3M PRIBOR-LIBOR differential   3M ex post CZK/DEM rate change
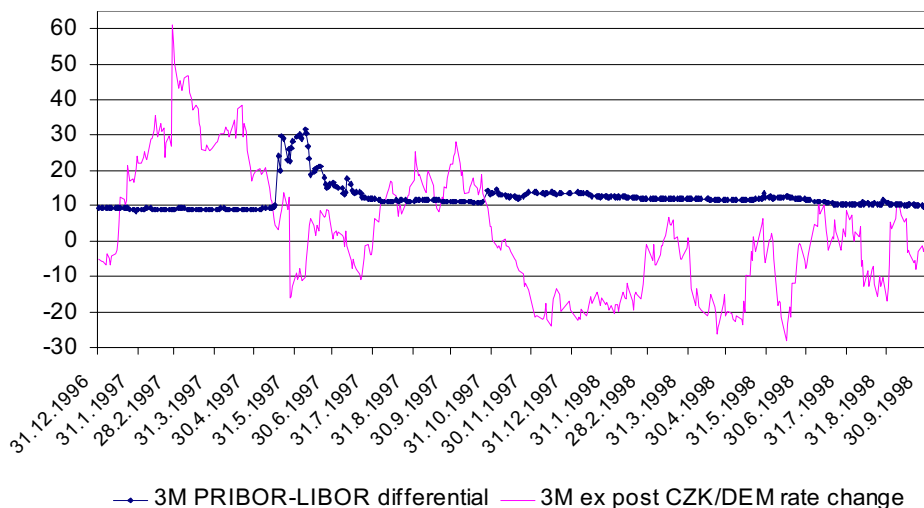
b) Six Month Interest Rates and Exchange Rate Changes

ex post exchange rate change on the interest rate differential is more often *negative.*[1]

The mentioned literature suggests that the worst results are generated by short money market rates, whereas the models that were able to improve on the original UIP-failure, were using other asset return measures. Specifically, the results improved along with the used instrument maturity. But, with long maturities, the choice of the instrument and the rate to be used becomes non-trivial. In this respect, deposit rates perform worst. On the other hand, no alternative long-horizon instruments have zero-coupon property, forcing one to replace the standard interest rate with the total return rate. Therefore, attempts are known to construct artificial measures of both return and time horizon in the UIP test, such as duration in place of maturity (see Alexius, 1998) or rates of return implied by the synthesized yield curve (see Meredith and Chinn, 1998). In these cases, estimation results give the "right" signs and sometimes do not reject the uncovered parity as such. The said ad hoc transition from *interest rates* to the *rates of return* leads to the same generalized uncovered return parity condition as I derive below on theoretical grounds.[2] Recently, there have appeared empirical papers that address the uncovered exchange rate parity for the long term bond yields directly (see e.g. Nadal-de-Simone and Razzak, 1999, or Berk and Knot, 2001).

In view of the above mentioned, the natural working hypothesis is that the uncovered parity is a property of properly defined returns on a particular pair of financial

---

1) In his own model, McCallum explains the "wrong" sign in this regression by the role of interest rates in the monetary policy reaction function.

2) The majority of traditional open macroeconomy models, including the Mundell-Fleming one, tacitly lean upon the parity between the return differential and the exchange rate change, even if it is formally called interest rate parity. Indeed, as soon as one aggregates the domestic investment possibilities in one composite asset and the foreign – in another, the two parameters which carry the name of interest rate become, actually, the total return rates on the resulting perpetuities. Therefore, they shall not be confused with the money market rates, whose highly specific role is usually played down in the textbook analysis.

instruments. In this paper, this hypothesis is verified by means of a micro-theoretically founded statement about the conditionally expected difference of *total returns* on any pair of comparable assets, paid out in two different currencies. This generalized uncovered total return parity condition is both free of the Jensen inequality effect and fares better empirically than the textbook UIP.

The paper is structured as follows. Section 2 outlines the intuition behind the uncovered parity of total returns with the help of a two-country, two-asset consumption and investment monetary model in discrete time, and discusses the properties of individual optimization that make Siegel's paradox irrelevant. Section 3 is dedicated to the stochastic portfolio optimization model in continuous time underlying GUTRP, its solution and formulae for the equilibrium asset prices, generalizing the traditional international CCAPM. I state the GUTRP equation for the stochastic log-exchange rate differential. Section 4 deals with the financial instruments and data used in empirical verification of GUTRP and reports on the confrontation of GUTRP with the CZK vs. the European Monetary Union (EMU) exchange rates as well as the USD vs. EMU rates. It also provides results of some elementary tests of UTRP.

2. Asset Return Parity in a Discrete Time Optimization Model

The first intuitive idea about the right formulation of the uncovered parity for the nominal exchange rate can be gained from a discrete time monetary optimizing model of a two-country world, with the models of Lucas (1982) and Obstfeld and Rogoff (1983), as the main prototypes.[3]

Let the period utility of the domestic agent be a function of real money balances $l = m/p$ and consumption rates: $u = u(l,c,c^*)$, with the usual growth and concavity properties of Sidrauski's (1967) model. Here, c and c^* are the consumption rates of domestic and the foreign good, p be the domestic price level and m denotes the private domestic nominal money holdings.

The agent can invest in an aggregate domestic asset paying nominal cash dividend d_t and an aggregate foreign asset paying nominal dividend expressed in foreign cash, d_t^* (both at date t). The secondary market prices of the assets at date t are P_t and P_t^*, respectively. S_t will denote the nominal price of foreign cash in domestic terms (nominal exchange rate) prevailing at date t. The agents are price takers in all markets. I define the current date $t + 1$ yields on the domestic asset, the foreign asset in foreign cash and the foreign asset in domestic cash by

$$y_t = \frac{d_t + P_t - P_{t-1}}{P_{t-1}}, \quad y_t^* = \frac{d_t^* + P_t^* - P_{t-1}^*}{P_{t-1}^*}, \quad 1 + y_t^i = \left(1 + y_t^*\right)\frac{S_t}{S_{t-1}}$$

Let $u_z(t)$ denote the partial derivative of the period utility with respect to argument z ($z = l, c, c^*$) calculated for the period t arguments. Further, let $R_{t+1} = \dfrac{p_t}{p_{t+1}} \dfrac{\beta u_c(t+1)}{u_c(t)}$ be the time t real price of one consumption unit at time $t+1$ (the inverse of the real domestic rate of return), and denote by E_t the expectation conditioned on the generally available information on date t.

In equilibrium, the solution of the agent's optimal consumption and investment problem in the spirit of discrete time CCAPM (cf. Breeden, 1979, or Ross, 1976),

---

3) Details of the model and its solution are available from the author on request.

implies the validity of the "international CCAPM", *regardless of the specific form of utility, supply side or statistical properties of uncertainty*:

$$E_t[1+ y_{t+1}] = E_t\left[\frac{S_{t+1}}{S_t}\left(1+ y^*_{t+1}\right)\right] + \frac{Cov_t\left[y_{t+1} - y^i_{t+1}, R_{t+1}\right]}{1 - \dfrac{u_l(t)}{u_c(t)}} \tag{1}$$

The deterministic variant of the model yields a simplified version of (1):

$$1+ y_{t+1} = \frac{S_{t+1}}{S_t}\left(1+ y^*_{t+1}\right) \tag{2}$$

Both (1) and (2) indicate that the right measure of interest on financial instruments traded on the secondary market, governing the exchange rate change in the parity formula, is the *total return on these instruments*.

The covariance term in (1) is difficult to analyze without knowing a closed-form solution for the optimization problem. However, the uncovered parity relation implied by (1) can serve as a heuristic hint as to what is wrong with the traditional formulations of UIP. Namely, (1) and (2) predict an uncovered parity between total returns/ current yields of specific pairs of assets, not between interest rates on the inter-bank market. Moreover, equation (1) and its continuous time analogues discussed in the next section, admit a non-zero country risk premium and a difference in total returns between two countries. The model also sheds light on the nature of the national asymmetry phenomenon leading to Siegel's paradox.

Recall that Siegel (1972) observed that UIP, applied without a narrower qualification of instruments and rates of return in a naive non-arbitrage way, cannot be valid simultaneously for international and domestic investors. Indeed, UIP amounts to the claim that from the domestic and the foreign investor perspective, respectively, the interest rates at home, i, and abroad, i^*, between times t-1 and t satisfy the relations

$$\frac{1+ i_t}{1+ i^*_t} = E_{t-1}\left[\frac{S_t}{S_{t-1}}\right] = \frac{E_{t-1}[S_t]}{S_{t-1}}, \ \frac{1+ i^*_t}{1+ i_t} = E_{t-1}\left[\frac{S_{t-1}}{S_t}\right] = S_{t-1}E_{t-1}\frac{1}{S_t} \tag{3}$$

(It is assumed that the information structure of the internationally integrated forex market is the same for the residents of all countries.) Then the two parity conditions together imply the equality $E_{t-1}\left[\dfrac{1}{S_t}\right] = \dfrac{1}{E_{t-1}[S_t]}$. However, unless the environment is perfectly deterministic, the left-hand side of the latter must be strictly greater than the right-hand side, as follows from Jensen's inequality (function $x \mapsto \dfrac{1}{x}$ is strictly convex; sometimes, Siegel's paradox is referred to directly as the Jensen inequality effect in the expected exchange rates). Therefore, the discussed general form of the uncovered interest rate parity cannot hold simultaneously for investors living at home and abroad.

The naive no-arbitrage formula (3) does not just suffer from the Jensen inequality effect. As such, it is incompatible with the two-country optimizing model, if the latter is extended to include nominal one-period domestic and foreign bills with rates of return i and i^*. Indeed, suppose that these nominal interest rates are exogenous (e.g. determined by the monetary authority). It is necessary to specify whether the access to these bills is restricted or not. If yes, the bill holdings cannot be

part of an internal solution to the optimization problem: they are optimally set at the minimal allowed negative level if the interest rate is low and at the maximal allowed positive level if it is high. According to the Kuhn-Tucker theorem, the parity for i and i^* must include non-zero Lagrange multipliers of the corresponding tight quantity constrains, making it different from (3).

If the access to bills is subject to a non-linear restriction (e.g. an endogenous subscription fee or a transaction cost with a non-linear dependence on the acquired/issued quantity), then the internal solution for the investment problem exists, but renders a parity condition different from (3). Subscription prices would lead to a formula analogous to (1), while the transaction cost approach (which is taken in the next section) leads to conventional supply and demand schedules for any security including the nominal bill. The latter approach gives rise to the generalized total return parity, which is free of Siegel's paradox.

For a more profound analysis of the total return parity it is convenient to resort to a continuous time portfolio optimization model, which offers a number of technical advantages in the treatment of uncertainty. In particular, the dynamics of asset prices can be calculated with the help of Itô's lemma.

3. Generalized Total Return Parity in the Shadow Asset Pricing Model

This section introduces a dynamic stochastic model of international investment and consumption in continuous time, with domestic and foreign liquidity arguments in the utility function in the role of liquidity constraints. The model is solved by means of a stochastic maximum principle including the adjoint equations. (The technique, pioneered in Bismut, 1976, and Hausmann, 1981, was further developed in Peng, 1990, and applied to the portfolio problem by Cadenillas and Karatzas, 1995. The present paper uses the approach of Derviz, 1999.) Supplies and demands for securities are derived in terms of their shadow prices (co-state processes of the optimization problems of the agents). The diffusion dynamics of the shadow price processes are given in terms of utilities, asset returns and their growth/attrition rates. The obtained differential equations for the shadow prices lead to Itô equations for the equilibrium asset prices, including the exchange rate. For the assets traded on a perfectly internationally integrated, highly liquid and nearly frictionless market, the GUTRP formula we obtain becomes very close to the uncovered parity statement suggested in Section 2 above.

3.1 Continuous Time Two-country Consumption, Investment, and Asset Pricing

The agents in the economy are identical households consuming a homogenous domestic consumption good C^d and a homogenous foreign consumption good C^i (consumption rates are c^d and c^i). The households hold domestic cash and sight deposits M (real balances) in the amount x^0, international cash and sight deposits I in the amount x^i, aggregate domestic asset D (number of shares x^d), and aggregate foreign asset F (number of shares x^f). The agent's vector of state variables is $x = [x^0, x^i, x^d, x^f]^T$. The M-price of D is denoted by P^d, the I-price of F – by P^{*f}. Symbol M can be traded against C^d, I and D, while foreign goods and assets must be first exchanged for M before they can be transmitted to investment or consumption at home. Symbol I is understood as a synthetic international security aggregating those currencies which are held by the agents in the accounts at domestic banks. Therefore, the nominal exchange rate S means the price in nominal domestic terms of

this currency basket. Accordingly, $Q = S/P$ (the "real exchange rate") is the price of this very basket in terms of *real domestic balances M, P* being the current domestic price level.

There is a stochastic instantaneous rate of return $d\pi^0$ on M (meaning the nominal rate of return on the sight deposit component of M less the inflation rate), and the M-dividend rate $d\Gamma^d$ on D. Foreign liquidity I grows at rate $d\pi^i$. Asset D has the internal deterioration rate $d\pi^d$. (One can think of a random default rate, stock dilution, etc. Naturally, for some securities comprising D, such as government bonds, this rate is zero.) International asset $f \in F$ has the I-dividend $d\gamma^f$ and the internal deterioration rate $d\pi^f$. Cumulative growth and dividend variables π^0, π^i, Γ^d, γ^f, π^d and π^f are Itô processes that generate the uncertainty structure of the model. Let $d\pi^0 = \mu^0 dt + \sigma^0 dZ$, $d\pi^i = \mu^i dt + \sigma^i dZ$, $d\pi^d = \mu^d dt + \sigma^d dZ$, $d\pi^f = \mu^f dt + \sigma^f dZ$, $d\Gamma^d = \Delta^d dt + \varepsilon^d dZ$, $d\gamma^f = \delta^f dt + \in^f dZ$. The diffusion terms are spanned by a d-dimensional vector Z of standard mutually independent Brownian motions, generating the filtration $F = (F_t)t \in {}_{R^+}$ satisfying "the usual conditions". F_t is the time-t publicly available information. All the processes appearing in the sequel are Markov diffusions with basis Z.

The agent chooses the decision path $t \mapsto I_t = \left[c_t^d, c_t^i, \varphi_t^i, \varphi_t^d, \varphi_t^f \right]^T$, where the last three components are formed by foreign currency purchase (sale if negative) rate φ^i, *rate* of D-purchases/sales φ^d, and rate of F-purchases/sales φ^f.

Let $x^l = x^0 + Qx^i$ be the total amount of domestic and foreign liquidity held by the agent. The transaction costs in the asset markets are defined as follows. For $\varphi^i > 0$, the amount $\varphi^0 = j(x^l, Q\varphi^i)$ is subtracted from the M-account in exchange for φ^i purchased units of I ($\varphi^0 > Q\varphi^i$). For $\varphi^i < 0$, $-\varphi^0 = -j(x^l, Q\varphi^i) > 0$ is the increment in the M-account in exchange for $-\varphi^i$ sold units of I ($-\varphi^0 < -Q\varphi^i$). The difference $j(x^l, Q\varphi^i) - Q\varphi^i$, a smooth increasing convex function of the absolute transaction volume $|\varphi^i|$, is the transaction fee paid to an intermediary. This fee depends smoothly and negatively on the agent's solvency x^l: the wealthier the agent is, the easier it is for him to operate on the FX market. In aggregate one can also interpret x^l in the above formula as a measure of depth, or liquidity, of the corresponding financial market segment. Thus, the deeper the market, the lower the transaction costs. Zero transaction costs are impossible unless there is no trade at all: $j(x, \psi) = 0$ if and only if $\psi = 0$. For a given j, the *marginal transaction function* is defined as $\upsilon = 1/j_\psi = h_\varphi$ in the forex as a function of x and $\varphi = j(x, \psi)$ (subscripts denote partial derivatives).

Transaction functions j^d and j^f on the D- and F-markets are defined similarly. The marginal transaction functions will be shortened to $\upsilon^d = \upsilon^d(x^l, \varphi^d)$, $\upsilon^f = \upsilon^f(x^l, \varphi^f)$. For simplicity, I assume that the sole measure of liquidity is the same cash holding value $x^l = x^0 + Qx^i$, i.e. the richer the agent is, the easier it is for him to trade on all asset markets.[4]

Under the above notations, the state-transition equations take the form

$$dx^0 = x^0 d\pi^0 + x^d x \Gamma^d - c^d dt - j(x^l, Q\varphi^i)dt - P^d j^d(x^l, \varphi^d)dt, \quad (4a)$$

$$dx^i = x^i d\pi^i + x^f d\gamma^f - c^i dt + \varphi^i dt - P^{*f} j^f(x^l, \varphi^f)dt, \quad (4b)$$

$$dx^d = -x^d d\pi^d + \varphi^d dt, \quad (4c)$$

$$dx^f = -x^f d\pi^f + \varphi^f dt \quad (4d)$$

---

4) Unless one models an economy with currency substitution, it is not necessary to distinguish between domestic and foreign cash holdings as the measure of the agents' ability to transact.

The household debt in either domestic or foreign liquidity is limited due to the dependence of the period utility function on x^0 and x^i. I am assuming the period utility to be the function $(x^0, x^i, c^d, c^i) \mapsto u(x^0, x^i, c^d, c^i)$, strictly increasing and concave in each argument. Partial derivatives of u with respect to x^0 and x^i in the negative domain are decreasing quickly enough to penalize increasingly negative cash holdings by the utility falling quickly to minus infinity (mimicking the cash in advance constraint). In this way, I prohibit explosive negative positions in M and I. In the positive domain, concavity of u ensures a diminishing marginal utility of increased liquidity holdings. Dependence of u on the two consumption variables must satisfy the usual Inada conditions.

The representative agent maximizes

$$E\left[\int_0^\infty e^{-\vartheta t} u(x_t^0, x_t^i; c_t^d, c_t^i) dt\right],$$ (5)

subject to the state-transition equation (4), the initial asset values given.

The solution of problem (4), (5) can be characterized by dynamic programming methods. Beside that, traditional consumption and portfolio optimization problems (see Merton, 1991) can be often analyzed by means of the "martingale" technique (see Duffie, 1992). The problem discussed here contains the state process in the utility function and, therefore, cannot be treated with the latter method. Instead, the technique based on adjoint equations is used.

The Shadow Price Solution of the Individual Optimization Problem

The maximum principle for the problem (4), (5) can be formulated in terms of the *shadow price vector* ξ, consisting of four shadow prices, ξ_0, ξ_i, ξ_d and ξ_f of asset categories M, I, D and F. These are adjoint processes of the problem (continuous time analogues of the Lagrange multipliers for the state-transition equations (4)). The laws of motion of the shadow prices are

$$d\xi_0 = \xi_0\left(\vartheta\, dt - d\pi^0 + \left|\sigma^0\right|^2 dt + j_x(x^I, Q\varphi^i) dt + P^d j_x^d(x^I, \varphi^d) dt\right) + \xi_i P^{*f} j_x^f dt - u_{x^0} dt,$$ (6a)

$$d\xi_i = \xi_0 Q(j_x + P^d j_x^d) dt + \xi_i\left(\vartheta\, dt - d\pi^i + \left|\sigma^i\right|^2 dt + Q j_x(x^I, Q\varphi^i) dt + Q P^f f_x dt\right) - u_{x^i} dt,$$ (6b)

$$d\xi_d = \xi_0\left((\sigma^0 - \sigma^k)\cdot(\epsilon^d)^T dt - d\Gamma^d\right) + \xi_d\left(\vartheta\, dt - d\pi^d + \left|\sigma^d\right|^2 dt\right),$$ (6c)

$$d\xi_f = \xi_i\left((\sigma^i - \sigma^f)\cdot(\epsilon^f)^T dt - d\gamma^f\right) + \xi_f\left(\vartheta\, dt + d\pi^f + \left|\sigma^f\right|^2 dt\right),$$ (6d)

and an appropriate transversality condition for ξ must be added to (6).

These adjoint equations apply to both domestic and foreign agents and are symmetric with respect to the country of residence, except for the national liquidity preferences u_{x^0}, u_{x^i} that need an obvious formal correction. This symmetry property resolves Siegel's paradox, as will be discussed in Subsection 3. 2.

First Order Conditions and Equilibrium Asset Prices

Optimal transactions in the forex, i.e. M/I-market, are described by the first order condition of optimality $\xi_i - \xi_0 Q j_\psi(x^I, Q\varphi^i) = 0$. This leads to the following expression

for the real exchange rate Q as the optimal reservation price (the inverse I-demand function if $\phi^i > 0$, and the inverse I-supply function if $\phi^i < 0$) for a domestic investor in the FX market:

$$Q = \frac{\xi_i}{\xi_0} j_\psi\left(x^I, Q\phi^i\right) = \frac{\xi_i v\left(x^I, \phi^0\right)}{\xi_0} \qquad (7)$$

The first order conditions for optimal transactions ϕ^d and ϕ^f are analogous to (7):

$$P^d = \frac{\xi_d}{\xi_0} v^d\left(x^I, \phi^d\right), \ P^{*f} = \frac{\xi_f}{\xi_i} v^f\left(x^I, \phi^f\right) \qquad (8)$$

In equilibrium, one can impose the market clearing conditions $\phi^d = \phi^d = 0$, $\phi^f = \phi^f = 0$ as well as the FX market clearing: $\phi^i = 0$. According to our assumptions, this means $v \equiv v^d \equiv v^f \equiv 1$ for all d and f. The asset market equilibrium is characterized by the following special case of (8):

$$\xi_0 = \frac{\xi_d}{P^d}, \xi_i = \frac{\xi_f}{P^{*f}} \qquad (9)$$

Conditions (9), derived here for the model with non-trivial liquidity constraints, are related to the continuous-time CCAPM (see Duffie, 1992). Namely, (9) comprises two "extended CCAPM's", one for domestic and the other for international securities. Tied together by (7), they render *a generalized international CCAPM*. Thanks to the explicit laws of motion of the shadow prices, one can decompose the risk premia that determine differences in asset returns. Next, (6), (7) and (9) will be used to derive the equilibrium dynamic of the exchange rate in relation to the return rate differential on liquid assets at home and abroad.

3. 2 *The Generalized Uncovered Asset Return Parity for the Expected Exchange Rate*

From now on, the real exchange rate Q is replaced by the nominal exchange rate S. Understanding of all variables and equations shall be adjusted accordingly. This replacement can be justified by our focus on international investors who do not care about domestic inflation. From a purely technical point of view, going over from real to nominal values of return rates and relative price changes leads only to changes in the diffusion parameters. The latter will not be analyzed explicitly. For arbitrary external supply in the forex (i.e. without the restriction $v \equiv 1$), (7) and (9) render the following pricing equalities:

$$\xi_0 = \frac{\xi_d}{P^d} = \frac{\xi_f v}{SP^{*f}}, \xi_i = \frac{\xi_f}{P^{*f}} = \frac{\xi_d S}{P^d v} \qquad (10)$$

Rewriting (10) as $\xi_d SP^{*f} = \xi_f P^d v$ and applying Itô's lemma together with (6b), (6d), one arrives at the Generalized Uncovered Total Return Parity formula

$$\frac{d\Gamma^d}{P^d} + \hat{\partial}P^d - d\pi^d = \frac{d\gamma^f}{P^{*f}} + \hat{\partial}P^{*f} - d\pi^f + \hat{\partial}S - \hat{\partial}v + h\left(P^d, P^{*f}, \frac{\phi^0}{x^I}\right)dt \qquad (11)$$

Here, h is shorthand for the term containing diffusion vector norms and scalar products that are not analyzed in detail.

The left hand side of this equation is the total return rate (instantaneous yield) dY^d on asset d: the dividend/coupon rate relative to the current price, plus the relative capital gain/loss, minus the internal value loss/attrition rate. In the same way,

the first three terms on the right hand side form the total return rate dY^f on asset f. The last two terms, which I shall denote $d\Lambda^{df}$, originate in transaction costs and risk-adjustment. Collectively, these terms can be dubbed the *disparity rate* between assets d and f. This name points at the fact that in the absence of $d\Lambda^{df}$, (11) would reduce to the standard uncovered total return parity condition on assets d and f.

Due to the disparity terms, the international CCAPM leading to (11) is able to explain violations of the textbook uncovered parity. Analogous results obtained by the martingale technique can be found in Zapatero (1995).

Equation (11) shall be interpreted in the following way. Fundamental information on the currency value is contained in the price/return on the domestic asset. The opportunity cost for the international investor of investing in domestic instruments is given by the foreign asset return. Volatility of domestic fundamentals and their correlation with the relevant foreign fundamentals is contained in variable h, which is constant in the simplest cases.

As a theoretic check of the GUTRP formula, let us show that Siegel's paradox does not arise for assets D and F. Exactly speaking, if the d-f disparity Λ^{df} and the f-d disparity Λ^{fd} are defined by (11), then they satisfy the necessary symmetry/consistency condition $d\Lambda^{fd} = -d\Lambda^{df} - |\rho|^2\, dt$, so that the equalities

$$dY^d = dY^f + \hat{d}S + d\Lambda^{df}, \; dY^f = dY^d + \hat{d}\left(\frac{1}{S}\right) + d\Lambda^{fd}$$

hold simultaneously. (For a strictly positive Itô process x, I use the shorthand $\hat{d}x = \dfrac{dx}{x}$.)

The trivial proof utilizes the shadow asset price method. The relation between dY^d and dY^f was obtained from (10). The first equation in latter is the fundamental asset pricing formula from the domestic investor perspective, whereas the second equation corresponds to the foreign perspective. Now observe that from the international investor vantage point not only S becomes $\dfrac{1}{S}$, but also v becomes $\dfrac{1}{v}$. Indeed, the marginal transaction process υ admits a symmetrical definition. It is the derivative of the other country cash with respect to the own country cash involved in the forex transaction: $v = \dfrac{d\psi}{d\varphi}$.

It follows that (11) must be just as invariant under the change of perspective from domestic to foreign as the equation from which it was derived, i.e. (10). This concludes the proof. The disparity equation $d\Lambda^{fd} = -d\Lambda^{df} - |\rho|^2 dt$ means that, differently from the full asset pricing formulae (10) and (11), the perceived forward exchange rate premium does not have to be symmetric. That is, even if there is no forward premium for domestic investors buying foreign currency, there is a non-zero premium for the foreign investors buying domestic currency. This is true as long as the exchange rate has a non-zero volatility ρ.

4. Empirical Evidence on the Asset Return Parity for the Exchange Rate

4. 1 *Discrete Time Representation of the Generalized Uncovered Parity*

As shown below, empirical properties of the total return parity exhibit a significant improvement compared to the traditional UIP. However, to be able to fully exploit the advantages of the GUTRP concept, one must find the proper procedure of

translating the abstract Itô calculus statements of the previous section into the language of discrete time financial market data. The present subsection contains a suggestion of how this can be done.

Derivation of the discrete time counterpart of the GUTRP equation (11) depends on the properties of utility and transaction functions defined in 3.1. It is easiest to work with homogenous utility and transaction functions. Then, conditions can be formulated under which (11) takes a much simpler Uncovered Total Return Parity form, namely

$$dY^d = dY^f + \hat{d}S + h^0 dt, \tag{12}$$

where h^0 is a term containing scalar products of diffusion vectors. These conditions are:

a) markets for the involved securities clear at any given moment without external infusions or withdrawals;

b) the capital market segments where the domestic and the foreign assets are traded are highly liquid at all times;

c) the used pair of instruments has the longest possible maturity.

Instruments with long maturities are desirable, because there is a distortion of the asset pricing formulae in models with a finite horizon. (The model of Section 3 deals with infinite horizon problems. Further simplifications that will be considered next also heavily rely on the absence of finite horizon effects.) The shadow prices of assets in a finite T-horizon model may be decisively dependent on the expectations about the development at times after T. (This is an alternative way to see why the poor predictive power of UIP based on short interest rates should not appear as a surprise.) For these effects not to matter, T must be far enough from the present moment. In that case, the after T-effects are sufficiently discounted.

Note the formal distinction between the current/instantaneous yield that appears in (11) and (12), and the yield to maturity. This distinction disappears completely in continuous time setting of Section 3. Accordingly, it is negligible in all examples with high frequency (i.e. daily) data, and remains small for weekly and even monthly time steps. Therefore, in the present paper I use the two yield notions interchangeably.

To derive a testable discrete time analogue of (12), one needs to define discrete time analogue of the stochastic differential ds in practically computable terms. First of all, we must fix an averaging algorithm to represent an *ex post* measure of the expected value change of a highly volatile exchange rate time series. The transition from continuous to discrete time measure of the exchange rate change also requires pinning down an additional "free" parameter, which defines the amplitude of the infinitesimal step of the random walk part of ds.[5] Therefore, the discrete time equation for testing the Variant A-UTRP will have the form (the "hat" symbol for any variable y denotes its rate of change: $\hat{y}_t = \dfrac{y_t - y_{t-1}}{y_{t-1}}$):

$$Y_t^d = Y_t^f + a_1 \hat{S}_t + a_0 + \varepsilon_t, \tag{13}$$

with parameters a_0 and a_1 to estimate; ε is a zero-mean error term. One needs a proper process to capture the time series properties of the residual terms ε. The latter cannot be expected to be independent. Indeed, standard discretization of a continuous time semimartingale typically results in a series with ARMA properties. Moreover, (11) suggests that statistical properties of the error term are heavily de-

---

5) An Itô process can be obtained as a limit of random walks with trend, if the step of the walk converges to zero. However, to define the convergence procedure completely, one needs to know the constant ratio of the walk step and the time step sizes (the latter is also converging to zero).

pendent on the model (GUTRP property itself is not). In general, GUTRP does not provide sufficient information about the statistical properties of ε. The latter needs to be discovered either empirically or by making the model more specific with regard to the market clearing mechanism.

The basic message of (13) is that the processes $Y^d - Y^f$ and $s = logS$ are positively correlated. This statement turns out to have solid empirical support. That is, (13) renders an equilibrium appreciation or depreciation trend (corresponding to the drift parameter of the exchange rate process in the continuous time model). As we shall later see, most of the time the correlation with the yield differential is manifested in one of the two alternative ways (to be further discussed in the next subsections):

A) for a time series of moving average exchange rate changes over a fixed interval (e.g. 3 or 6 months),

B) for the time series of log-levels of the exchange rate itself over a fixed period, usually several months long.

In general, smoothing over a number of months (in Variant A) works best for mature forex markets outside the periods of major expectation revision, whereas possibility B seems to be typical for "young" currencies or the "old" ones when opinions are being newly formed.

4. 2 *Instruments and Data*

As follows from condition c), the best instruments for the application of GUTRP would be perpetuities, e.g. common stock double floated internationally as GDR. Unfortunately, this approach would work poorly for most transitive economies, partly for reasons related to conditions a) and b). For similar reasons, there could be a problem of utilizing emerging market corporate bonds or eurobonds. Therefore, at the moment, the domestic asset in the GUTRP equation is best represented by one of the long-horizon government bonds. These bonds have small individual risk premium, and the liquidity of the corresponding market segment is highest among all instruments with similar maturity and coupon regime, bringing them close to the ideal state described by condition b).

Total Asset Return Parity for the CZK/EUR Rate

The longest maturity of Czech government bonds available since 1997 has been 5Y (see Figure 2 for the sample period July 1, 1997 – December 31, 1999), which determined our first choice of the representative Czech instrument. Since the currency serving as the predominant "entry-exit" one for traders in Czech financial products is the DEM (EUR after January 1, 1999), the representative foreign instrument must be the German government 5Y bond.

CZK counterparts of the 10Y government bonds from Germany and other euro zone countries became available early in 1999. First, a 10-year bond in CZK was issued by the European Investment Bank. This instrument was used in our analysis of GUTRP until Spring 2000, when the first 10Y Czech government bond entered the market. The high correlation, i.e. a common direction of change of the corresponding yield differential with the nominal CZK/EUR exchange rate until April 2000 can be seen in Figure 3 (Variant B-UTRP). This fact alone is a sizeable improvement compared to the short rate differentials, where no co-movements could be found.[6]

---

[6] A similar exercise carried out for 10Y US-government bond yields did not confirm a simple parity relationship between CZK and USD. The probable reason is insufficient size of the market for direct CZK-USD spot transactions.

Figure 2

The Czech-German 5 Years Government Bond Yield Differential and the Log-nominal CZK/DEM Exchange Rate
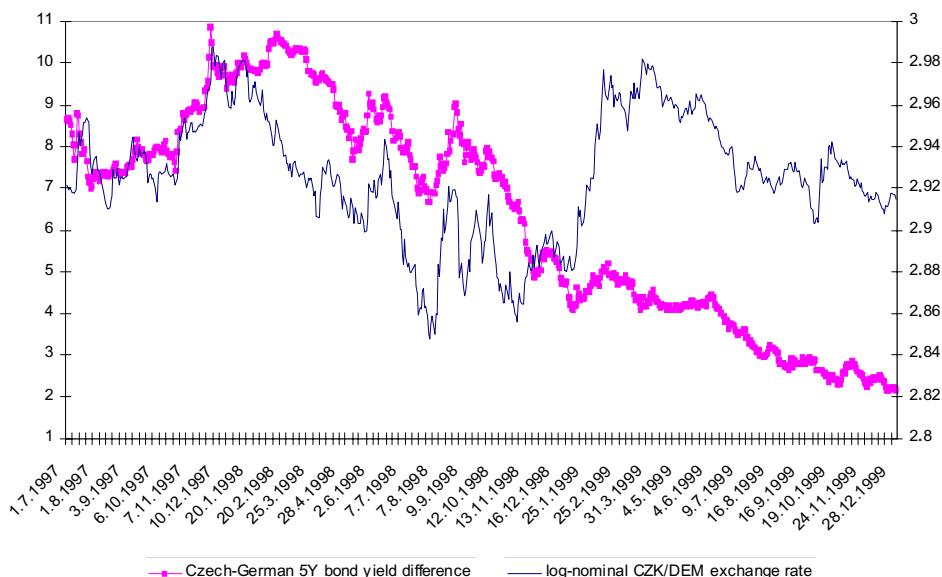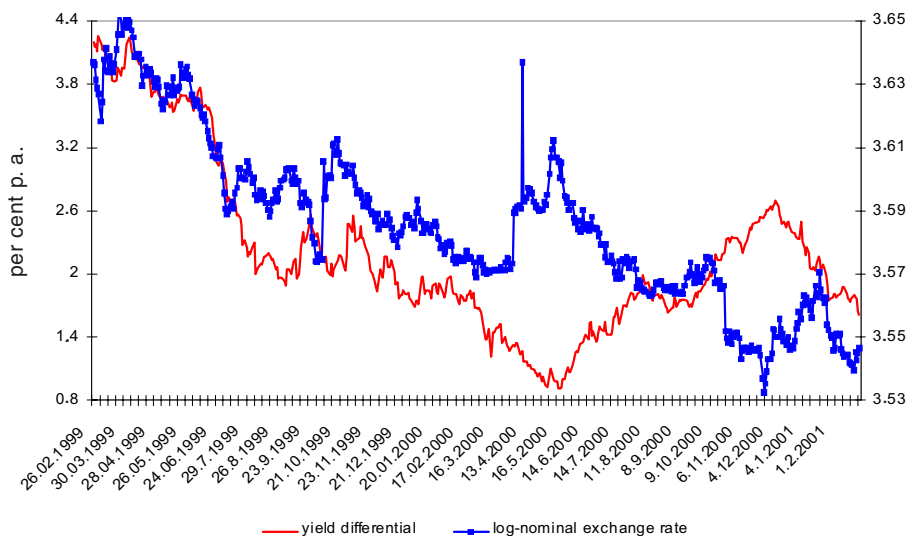

Czech-German 5Y bond yield difference — log-nominal CZK/DEM exchange rate

Figure 3

The Czech-German 10 Years Government Bond Yield Differential and the Log-nominal CZK/EUR Exchange Rate

yield differential — log-nominal exchange rate

Close correlation of *S* and the yield differential predicted by GUTRP, except for some short episodes of variable disparity, can be also found for the 5Y yield bonds, on a longer sample between 1997 and 1999. Especially significant are co-movements during the second half of 1997, i.e. the half-year immediately following the

currency crisis. The Czech money market rates of that period were characterized by a difficult recovery from the interventions and liquidity squeeze of May and June the same year. At the same time, yields of long maturity bonds already signaled the return to normality and even predicted the exchange rate movements correctly. As Figure 2 indicates, qualitative incidence of the short-term shifts in the Czech-German yield differential and the CZK/DEM nominal exchange rate is almost perfect. That is, ups and downs of the former are almost always accompanied by the tilts of the latter in the same direction (described above as Variant B). However, the magnitude of these movements can vary considerably, so that the distance between the two schedules indicates a slowly changing disparity term over time.

Quantitative estimates require smoothing of the exchange rate change series, since the original process $\tilde{S}$ volatility is very high. The smoothing procedure utilized here is the following. We take the average slope of (i.e. regress on the time step variable) the log-nominal CZK/DEM or CZK/EUR exchange rate curve, starting on the current day and ending on the final day of the time interval over which averaging is required. This moving slope value is the desired average future relative change of the currency price. Here, we experiment with averaging over 3 month and 6 month intervals, for the CZK/EUR rate. The outcome for the period between February 26, 1999 and February 22, 2001 is shown in Figure 4. (For obvious reasons, the said slope series break off 3 and 6 months earlier.)

Figure 4
The Czech-German 10 Years Bond Yield Differential and the 3M and 6M Log-nominal CZK/EUR Exchange Rate Slope
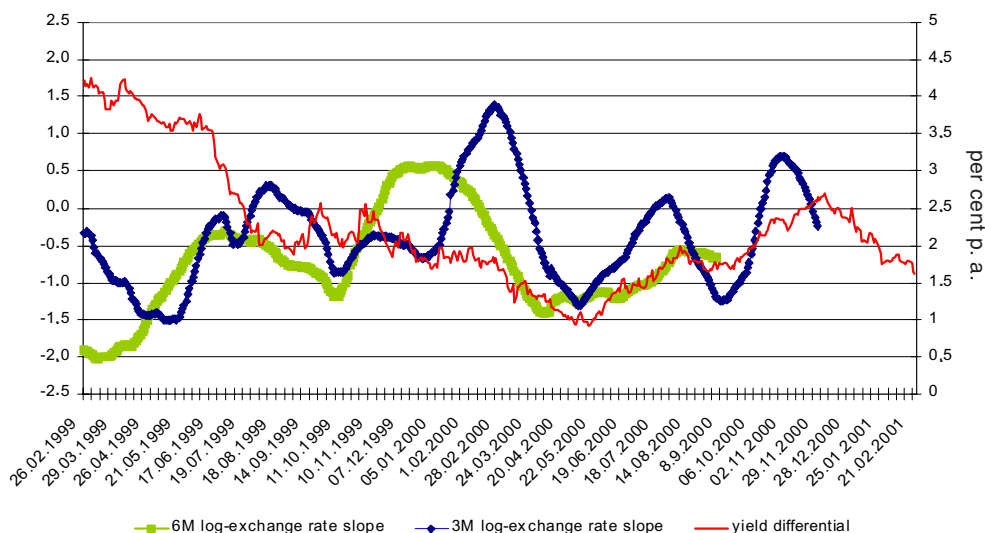

Figure 4 suggests a strong positive correlation between the ER-change and the yield differential occurring between mid-1999 and the third quarter of 2000 (Variant A-UTRP) for both 3M and 6M ER-change series. The single apparent deviation appears in the 3M-series in Spring 2000 and can be attributed to the massive intervention of the Czech National Bank against strengthening CZK in April of that year. Even more importantly, the second half of 1999 witnessed the transition of the CZK market from Variant B-parity to Variant A-parity. This fact may be a sign of maturity of the corresponding market segment, since Variant A is most frequently observed for major world currencies. According to this conjecture, the second half of 1999 might be the time when Czech

government bonds and cash "joined the club" of regular internationally traded assets. Its behavior has been even closer to Variant A-UTRP than that of the USD/EUR rate (see Figure 5).

Figure 5
USA–EMU 10 Years Government Bond Yield Diffferential and the 3M and 6M USD/EUR Log-exchange Rate Slope

6M log-exchange rate slope · 3M log-exchange rate slope · yield differential
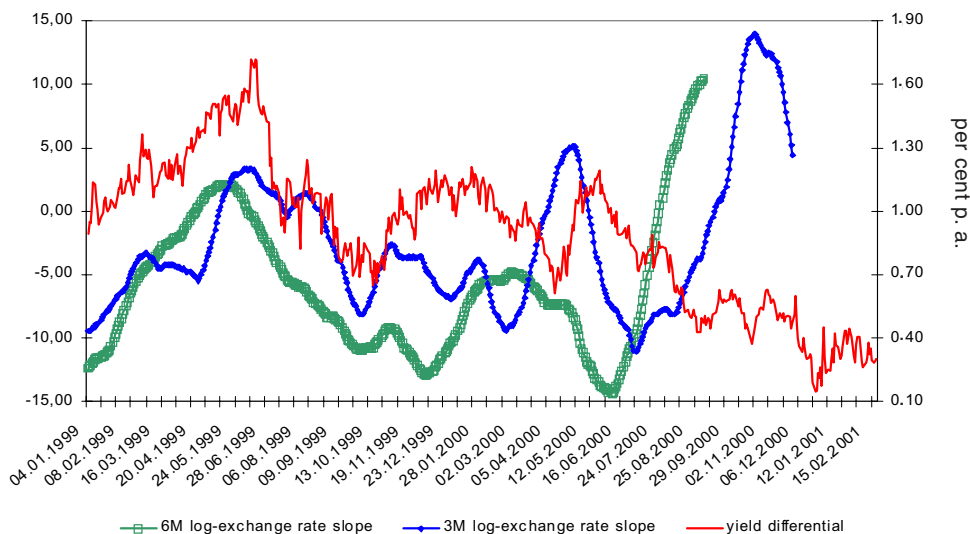
The USD/EMU Exchange Rates

The UTRP property for the USD exchange rates of the EMU currencies is most naturally analyzed with the help of the DEM/USD rate. Alternatively, it suffices to analyze UTRP for the Austrian schilling (ATS), a currency that had been effectively pegged to the DEM for many years preceding the introduction of the euro. Also the benchmark Austrian government 10Y bond yields were closely tied to the German ones in recent years. Arguably, conditions a) – c) of Subsection 1 are fulfilled for the Austrian-U.S. 10Y bond pair quite satisfactorily. Due to the availability of high quality Austrian bond yield data, I choose the latter possibility.[7] Figure 6 displays the corresponding sample of daily data between January 1, 1994 and February 22, 2001 together with the log-nominal ATS/USD exchange rate. First of all, one notices that the correlation between the log-exchange rate *level* and the yield differential, i.e. the Variant B-parity (see Figure 6a), cannot be expected to hold but for a number of isolated periods. The most recent of them started in mid-2000 and ended in the first weeks of 2001.

The ten year yield differential and the *ex post* smoothed 3M change in the ATS/USD exchange rate are featured in Figure 6b (I used the same moving 3M log-ER slope statistic as for the CZK/EUR rate). The immediate conclusion is the prevalence of a Variant A UTRP-conform behavior. However, the degree of disparity is different during different periods of UTRP-validity, interrupted by shorter episodes of the
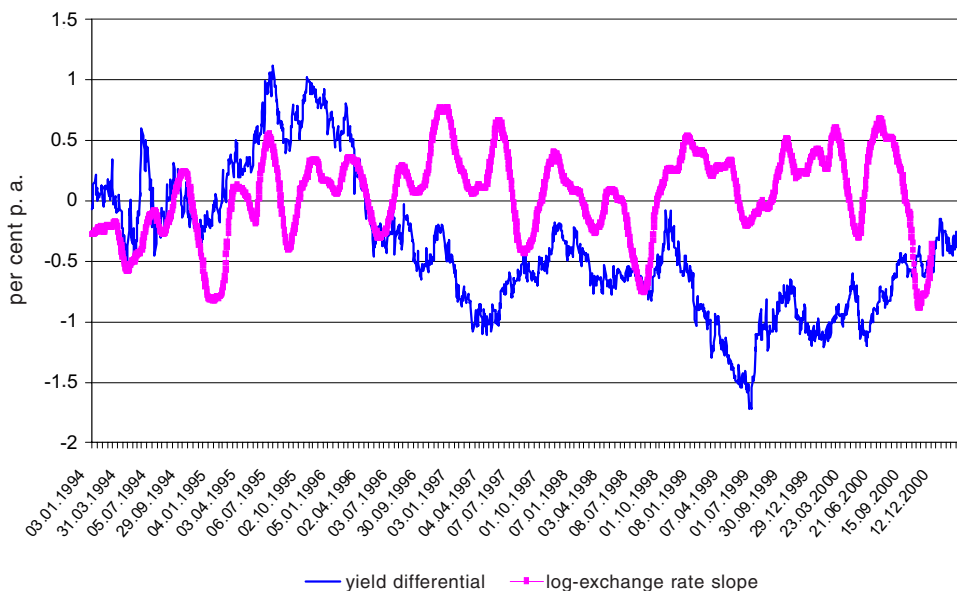
Figure 6
10 Years Government Bond Yield Differential between Austria and USA and the ATS/USD Exchange Rate

(a) Logarithm of the Nominal Exchange Rate
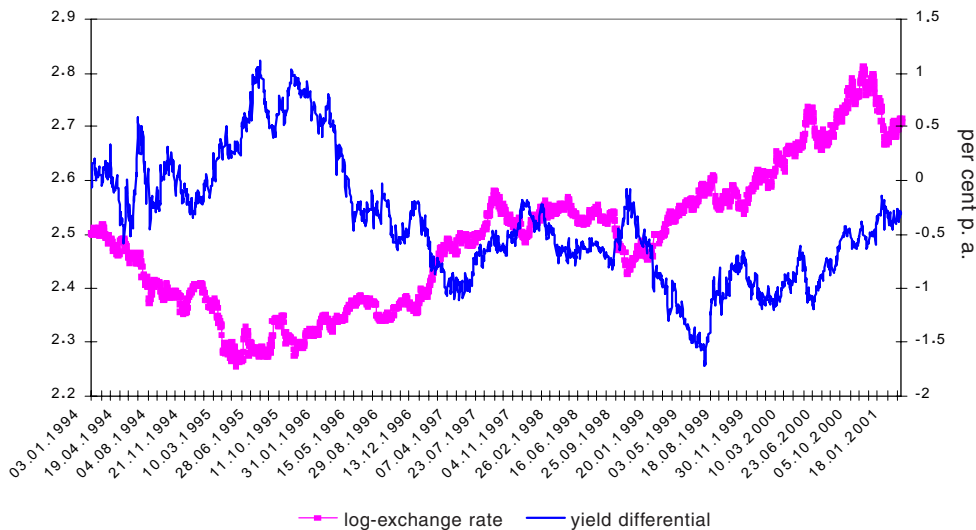
(b) 3M Log-exchange Rate Slope Values

country premium revision. Sometimes, these episodes mean complete violation of the uncovered parity in either Variant A or B (e.g. the second half of 1996). More often, a breakdown of uncovered parity in the smoothed ER-change sense (Variant

A) is offset by its validity in the log-ER level sense (Variant B, examples are the first half of 1997 and the second half of 2000). A comprehensive theory explaining this phenomenon would exceed the scope of the present paper. The first intuitive hypothesis might be a difference in the typical holding times by international investors of government bonds of a developed country at "no-surprises" times, and of the same country at times of expectation-revision, when the bond holding times are shorter and less regular. The deviation from Variant A-UTRP that became evident starting in October 2000, may indicate the protruded period of micro-heterogeneity of forex traders in the USD/EUR pair, be it for information or liquidity management reasons.

4. 3 *Regression Analysis*

In this subsection, I report on the outcome of elementary regression experiments for the Variant A-UTRP (smoothed ER-change measures vs. the yield differential) of CZK/EUR and USD/EUR rates. Additionally, an error-correction exercise for the Variant B-UTRP (log-nominal ER vs. the yield differential) of the CZK/DEM rate is described.

Variant A

Estimations were conducted on the 3M and 6M CZK/EUR rate changes vs. CZ-EMU 10Y yield differentials data in the July 1, 1999 – September 6, 2000 sample, when both visual inspection and formal correlation analysis indicated that the *ex post* parity was valid. Ordinary least squares regressions were run both without controlling for residual auto-correlation and for the ARMA(1,1) – residual assumption. (This is the most natural specification when Itô processes of the UTRP formula (12) are discretized.) The results are reported in Table 1. As we see, the ARMA(1,1) – specification for residuals reduces the significance degree of the exchange rate slope terms, but improves other diagnostics dramatically, at the same time leaving the constant term (standing for the country premium, or disparity) estimates in the same range.

Table 1
Estimation Results of the Yield Differential on the Log-exchange Rate Slope Regression, the CZK/EUR Rate (*t*-statistics in parentheses)

Equation	Constant	ER slope	Stand. error	R^2	F-stat.	DW-stat.	AR(1)	MA(1)
3M slope, white noise residuals	1.83 (69.65)	0.05 (3.86)	0.42	0.049	14.93	0.031		
6M slope, white noise residuals	1.98 (64.28)	0.11 (8.78)	0.38	0.21	77.13	0.037		
3M slope, ARMA(1,1) residuals	1.67 (10.27)	0.024 (0.86)	0.073	0.97	3182.3	1.99	0.87	-0.03
6M slope, ARMA(1,1) residuals	1.77 (10.78)	0.068 (1.55)	0.073	0.97	3200.1	1.99	0.97	-0.03

The same Variant A-UTRP for 3M and 6M USD/EUR rate changes vs. 10Y U.S.-EMU yield differential was tested on the sample January 4, 1999 – September 6, 2000. The results with and without the ARMA(1,1) – assumption for residuals are summarized in Table 2.

Table 2
Estimation Results of the Yield Differential on the Log-exchange Rate Slope Regression, the USD/EUR Rate (*t*-statistics in parentheses)

Equation	Constant	ER slope	Stand. error	R^2	F-stat.	DW-stat.	AR(1)	MA(1)
3M slope, white noise residuals	1.36 (76.60)	0.016 (5.43)	0.21	0.078	29.46	0.093		
6M slope, white noise residuals	1.32 (80.64)	0.11 (16.98)	0.16	0.45	288.48	0.157		
3M slope, ARMA(1,1) residuals	1.13 (11.14)	0.008 (0.85)	0.063	0.92	1307.6	1.97	0.97	-0.23
6M slope, ARMA(1,1) residuals	1.26 (13.12)	0.024 (2.19)	0.063	0.92	1319.0	1.97	0.96	-0.22

Adding the ARMA(1,1) – specification for residuals, we get even better results than in the CZK/EUR case as regards the significance of the exchange rate slope coefficients, at the same time improving other diagnostics. Also in line with the CZK/EUR case, the constant term estimates (the disparity) are close to the previously obtained ones.

The above results present promising evidence in favor of the UTRP concept for both studied exchange rates. We have not just established a significant correlation of the yield differential with the ER-changes in the first group of regressions (without the ARMA specification), but also confirmed the ARMA properties of residuals predicted by the theory, in the second regression group. The outcome is particularly reassuring when compared to the unsatisfactory results of all existing UIP-verifications conducted with the money market rates.

Variant B for the CZK/DEM Exchange Rate

Currencies and time periods for which the only tangible UTRP-evidence is in Variant B-form require the use of regressions of a less straightforward form than the Variant A case. The possibility to be exploited below is an error-correction equation for the log-exchange rate and the yield differential series. This is done on the sample July 1, 1997 – December 30, 1999. Separate unit root tests for $s = log\ S$ and $yd = Y^d – Y^f$ do not reject the I(1) – hypothesis at 95% – significance level for either of the series. Accordingly, I look for a long-term relationship between s and yd, particularly when the visual inspection (see Figure 2) suggests a co-movement of the two series, up to a single outstanding episode. Note that if the initial date of s-measurement is fixed arbitrarily within the tested time span, then its values at other dates get into a one-to-one correspondence with s-differences relatively to the cho-

sen starting point. This justifies the relevance of the co-integration of *yd* with *s*, and not Δ*s*, for the UTRP analysis. The equation to be estimated is

$$\Delta s_t = \gamma \Delta y_t + \lambda(s_t - \theta y_t) + A_t + \varepsilon_t \qquad (14)$$

The term in parentheses stands for the long-term relation between changes in *s relative to a fixed initial date*, and levels of *y*. Co-integration implied by (14) can be roughly associated with "smoothed expectations" understanding of Variant B-UTRP. That is, the error-correction equation provides another smoothing device for *s*-changes appearing in the GUTRP theorem.

Since the time series estimation issues associated with testing (14) are extensive and exceed the scope of the present paper, I provide only a few comments here. The main conclusion is that for the time span between July 1997 and December 1999, there is a powerful quantitative empirical support for the Variant B-UTRP for CZK/DEM and CZK/EUR rates for two identified sub-periods.

The important property of the sample featured in Figure 2 is a suspected break point in the data between 10 and 20 February 1999. Both before and after this short time interval, Variant B-parity of the CZK/DEM exchange rate with the 5Y Czech-German yield differential holds satisfactorily even at the first glance. Only in mid-February 1999 did the CZK depreciate against DEM much faster than would be justified by the yield difference. The break point conjecture was, indeed, validated by the asymptotic Wald test of coefficient equality in (14), applied to the data split into the parts before and after the week of 15-19 February 1999. Besides, separate estimation of the two sub-samples (including two runs for the 2nd sub-period, one with the 5Y and the other with the 10Y bond yields) confirmed the significance of coefficients in (14) and did not reject the independence of residuals.

The suddenly revised disparity for the CZK/DEM rate reflects an important structural change in the composition of traders on the CZK market. At the time, many speculators abandoned the CZK positions, no longer attracted by the arbitrage between the Czech and the eurozone interest rates that had existed previously. Apparently, the reduced amount of speculation after that withdrawal has led to both a general reduction in FX trade with the CZK and an activity ebbing on the Czech bond market. This conjecture is sustained by the reduction in volatility in (14) between the first and the second sub-period, expressed by a lower error variance. Symptomatically, the error variance outcomes do not differ much for the 5Y and the 10Y yield differential data, therewith enhancing the verisimilitude of the activity-reduction explanation of the break point.

5. C o n c l u s i o n

The paper studied the uncovered parity relation of the CZK/EUR exchange rate with the Czech-EMU return rate differential. It has proposed a micro-foundation for the uncovered exchange rate parity theorem for the total return differentials on representative national assets. The model should work best when applied to instruments traded on a liquid and low-friction secondary market and equally available to residents and non-residents. Since issuer-specific risk factors are to be minimized, the best candidates are long maturity government bonds. Thus, the appropriate assets for testing uncovered parity are different from the short maturity rates unsuccessfully employed by the traditional uncovered interest rate parity analysis. The theoretical result is stated as the Generalized Uncovered Total Return Parity formula. The latter is free from the Jensen inequality-type inconsistencies characteristics of the traditional UIP. Even its simplified form, the Uncovered Total Return Parity with a constant country premium, expected to hold only on very low-friction markets, finds

empirical support both for the CZK/EUR (CZK/DEM) and for the USD/EUR (USD/DEM) exchange rate. The utilized instruments are Czech, German and U.S. 5 and 10 years government bonds.

The UTRP condition can be used to extract information on the prevailing market sentiment regarding future movements of the exchange rate. Most of the time, information about the opinions of international investors on the expected appreciation or depreciation is reflected in the currently valid difference of national returns. This information includes the expectations about those fundamental variables, which the investors consider relevant for their domestic currency demand. The fundamentals influence the exchange rate formation through the secondary market yields of traded instruments.

Since the generalized UTRP follows from the Euler equations of the underlying optimization model, this property does not rule out multiple equilibria and bubbles in asset prices, including the exchange rate. Nevertheless, it supplies valuable information about the properties of the equilibrium exchange rate. For instance, if the disparity term has become non-stationary, this signals an advent of the impending asset market turbulence. In the background might be either a reduction in liquidity, or an external intervention affecting the volumes of traded instruments, or a sharp change in the statistical properties of fundamental variables. At times preceding pressures on the exchange rate correction, GUTRP may be able to indicate its direction and extent. This method has an advantage over other known methods of expectation-extraction from financial instruments in that it does not require the use of the risk-neutral probability. It can, therefore, be applied to incomplete asset markets.

References

Alexius, A. (1998), "Uncovered Interest Parity Revisited." Stockholm, Sveriges Riksbank, WP No. 53.
Berk, J. M., Knot, K. (2001), "Testing for Long Horizon UIP Using PPP-based Exchange Rate Expectations." *Journal of Banking and Finance*, 25, pp. 377-391.
Breeden, D. (1979), "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics*, 7(3), pp. 265-296.
Bismut, J. M. (1976), "Théorie probabiliste du contrôle des diffusions." *Memoirs of the American Mathematical Society*, 4(167).
Cadenillas, A., Karatzas, I. (1995), "The Stochastic Maximum Principle for Linear, Convex Optimal Control with Random Coefficients." *SIAM Journal of Control and Optimization*, 33(2), pp. 590-624.
Campa, J., Chang, K. (1996), "Arbitrage-based Tests of Target Zone Credibility: Evidence from ERM Cross-rate Options." *American Economic Review*, 86, pp. 726-740.
Cox, J., Ingersoll, J., Ross, S. (1985), "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, 53, pp. 363-384.
Derviz, A. (1999), "Adjoint Processes of the Portfolio Optimization Problem and Equilibrium Asset Prices." Prague, Institute of Information Theory and Automation, Research Report No. 1954.
Duffie, D. (1992), *Dynamic Asset-Pricing Theory*. Princeton, N.J.: Princeton University Press.
Duffie, D., Zame, W. (1989), "The Consumption-based Capital Asset Pricing Model." *Econometrica*, 57, pp. 1279-1297.
Engel, C. (1996), "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence." *Journal of Empirical Finance*, 3, pp. 123-192.
Froot, K., Thaler, R. (1990), "Anomalies: Foreign Exchange." *Journal of Economic Perspectives*, 4(3), pp. 179-192.
Hausmann, U. G. (1981), "On the Adjoint Process for Optimal Control of Diffusion Processes." *SIAM Journal of Control and Optimization*, 19(2), pp. 221-243.
Lucas, R. (1982), "Interest Rates and Currency Prices in a Two-country World." *Journal of Monetary Economics*, 10, pp. 335-359.

McCallum, B. (1994), "A Reconsideration of the Uncovered Interest Parity Relationship." *Journal of Monetary Economics*, 33, pp. 105-132.

Meese, R., Rogoff, K. (1983), "Empirical Exchange Rate Models of the Seventies." *Journal of International Economics*, 14, pp. 3-24.

Meredith, G., Chinn, M. (1998), "Long-horizon Uncovered Interest Parity." London, NBER, WP No. 6797.

Merton, R. (1991), *Continuous-Time Finance*. London: Basil Blackwell.

Nadal-de Simone, F., Razzak, W. A. (1999), "Nominal Exchange Rates and Nominal Interest Rate Differentials." Washington, IMF, WP/99/141.

Obstfeld, M., Rogoff, K. (1983), "Speculative Hyperinflations in Maximizing Models: Can we Rule them Out?" *Journal of Political Economy*, 91(8), pp. 675-687.

Peng, S. (1990), "A General Stochastic Maximum Principle for Optimal Control Problems." *SIAM Journal of Control and Optimization*, 28(4), pp. 966-979.

Ross, S. (1976), "Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*, 13(12), pp. 341-360.

Sidrauski, M. (1967), "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review*, 57(5), pp. 534-544.

Siegel, J. (1972), "Risk, Interest Rate and the Forward Exchange." *Quarterly Journal of Economics,* 86(II), pp. 303-309.

Söderlind, P., Svensson, L. (1996), "New Techniques to Extract Market Expectations from Financial Instruments." *Journal of Monetary Economics*, 40, pp. 383-429.

Zapatero, F. (1995), "Equilibrium Asset Prices and Exchange Rates." *Journal of Economic Dynamics and Control*, 19, pp. 787-811.