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The Reverse Banzhaf and Shapley – Shubik Index for SMEs in Slovak Medical Labs

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Abstract:

This study examines the potential for cooperation between traditionally competitive small and medium-sized enterprises (SMEs) in the sector 86901 - Medical laboratory services in Slovakia as a strategy to enhance competitiveness in the face of large market players. By building coalitions, SMEs can overcome natural disadvantages, such as limited economies of scale, a lack of capital, or limited access to innovation. In this article, we will focus on the possibilities of applying the Banzhaf and Shapley – Shubik indices, as well as their reverse interpretation, to quantify the individual strength of a player within potential coalition groupings, with an emphasis on Slovak SMEs. The quantification of the indices considers key regulatory elements, including compliance with antitrust laws, and these legal frameworks are crucial for promoting fair competition while enabling SMEs to reap the benefits of cooperation.

Keywords: cooperation, market share, strength indicators, game theory, Banzhaf index, Shapley – Shubik index

Jel Codes: C71, L13, L26, K21

1. Introduction

Small and medium-sized enterprises (SMEs) form the backbone of modern economies, particularly in the European Union, where they account for 99.8% of all businesses, employ approximately 88 million people, and generate 53.1% of the value added (€4.8 trillion). Despite their key role, SMEs face significant challenges in competing with large corporations,

which dominate markets due to economies of scale, better access to capital, and extensive market influence. Rising costs, limited resources, and difficult market access exacerbate these challenges, leading SMEs to seek innovative strategies to increase their competitiveness (Commision, 2024) (Beck, et al., 2005).

In addition to these factors, SMEs in the European Union also face a fragmented legal and tax environment, which makes it challenging for them to operate effectively within the Single Market. The reports "Much more than a market" and "The future of European competitiveness" draw attention to the fact that there is no truly single market within the European Union, as companies have to face different regulatory regimes in the areas of taxation, labour law and trade rules, which significantly reduces their competitiveness in the European area. To address these differences, the European Commission has proposed the introduction of a so-called 28th regime, which would provide a uniform legal environment for SMEs to do business across borders and simplify administrative obligations, including relevant aspects of corporate law, insolvency law, labour law and tax law (Letta, 2024) (Commission, 2024) (European_Commision, 2025).

One of the strategies employed by SMEs is to form cooperative coalitions, where actors collaborate to pool resources, reduce costs, and gain greater market power. Through such cooperation, they can achieve economies of scale and strengthen their market position, while retaining individual autonomy. However, such cooperation must operate within the limits of competition in order to avoid practices that could undermine fair market dynamics. European Union and its national regulators ensure that cooperation agreements do not lead to cartels, price fixing or other forms of unfair competition. These legal frameworks are crucial for promoting fair competition, while also enabling SMEs to reap the benefits of cooperation (Brandenburger and Nalebuff, 1996).

To avoid distortions of competition, the European Union sets clear rules for the admissibility of strategic alliances, in particular through merger control legislation. These rules require entities to analyse their combined market share before implementing mergers or alliances. If the thresholds set are exceeded, the European Commission assumes the decision-making power and in other cases, the evaluation remains the responsibility of the national regulatory authorities. This regulatory framework forms the key context in which the admissibility of coalition structures among SMEs is assessed (Auditors, 2020).

It is important to emphasise that cooperation strategies between SMEs do not aim to reduce competition, but rather to promote it. By forming alliances, SMEs can compete

together with dominant players, thereby increasing market diversity and stimulating innovation. Such dynamics ultimately benefit consumers through lower prices and better product offerings. The coalition behavior of subjects is studied within the framework of cooperative game theory. Small and medium-sized enterprises (SMEs) face significant competitive disadvantages vis-à-vis large market players, stemming from limited resources, lower economies of scale and limited access to distribution channels (Nagarajan and Sošić, 2007) (Beck, et al., 2005).

This article examines the potential for coalitions among SMEs. The creation of coalition groupings between SMEs can be a tool to counterbalance the market dominance of large enterprises, while simultaneously creating a framework for assessing coalition structures that are compliant with antitrust measures and promote fair competition. The study of coalition relations is a key subject in cooperative game theory. This article aims to point out the possibilities of using the Banzhaf and Shapley – Shubik index to estimate the coalition potential of enterprises in a specific sector in the Slovak Republic. The chosen computational approach enables the identification of undertakings that have the greatest flexibility in forming permissible coalitions and those that, conversely, are limiting elements from the perspective of competition. In contrast to the classical interpretation of these indices, we introduce them into a reverse form. It is the reverse interpretation that has its logic within the existing legislative environment, particularly antitrust laws, and evaluates the coalition potential of a player based on permissible groupings in a given sector. Selected enterprises operating in sector Medical laboratory services, classification of economic activities in the European Union - NACE classification 86901, will be analyzed based on data for 2022. Given that this sector has already been inspected by the Antimonopoly Office of the Slovak Republic based on data for 2022, we approach the analysis sensitively in this work, while our goal is not to encourage illegal forms of market coordination, but rather to provide an analytical tool that can help smaller companies compete more effectively with dominant players. We will present a methodological procedure that can help smaller companies to estimate better to estimate their own coalition potential better, but also the potential of their competitors and potential coalition partners in the existing legal environment so that the applicable competition rules are taken into account in terms of not exceeding the set market share. Such coalition groupings can bring benefits to SMEs, enabling them to compete more effectively with large players and offer a more advantageous product to end customers. Last but not least, an analysis of the coalition potential in a specific sector could be a valuable source of information for public administrations when assessing market concentrations, proposing

regulatory measures, and better targeting support for SMEs, which could create fairer conditions for business and increase the competitiveness of the entire sector.

The article is structured as follows: In the first part, we review the relevant literature in the field of cooperative game theory with an emphasis on the Banzhaf and Shapley – Shubik index as selected indicators of player strength. We also offer an overview of relevant legal standards in the field of competition protection of the European Union and the Slovak Republic. In the second part, we will formalize these indices based on standard literary sources, and we will also deal with their reverse interpretation. The central part of the article is the third part, where we analyze a selected sector in the Slovak Republic, specifically the Health Laboratory Services sector. The selected sector was chosen due to oits importance for public health, as well as its high market concentration, which raises competition issues. In addition, this industry has already been the subject of interest to the Antimonopoly Office of the Slovak Republic, which provides a suitable framework for the application of the proposed methodology and for verifying its practical applicability in real market conditions.

This approach provides a new way of quantifying coalition potential in a regulated economic environment, where traditional indices of decision-making power acquire a different interpretative value. While most existing studies focus on the political or academic applications of these indices, this article aims to demonstrate their practical application in the business sphere, specifically in assessing the admissibility of strategic cooperation among enterprises in light of the legislative limits on market concentration. The results achieved demonstrate that small enterprises can also play a key role in cooperative structures, if they have a high degree of flexibility without the risk of infringing competition rules.

The calculations were conducted using our own codes in the Python language. At the end of the article, we present the possibilities for further research in the field of analysis of the coalition potential of players.[1]

Firms with high market concentration can undertake merger & acquisition (M&A) if they meet the relevant regulatory requirements. A notable example is the merger of Holcim and Lafarge, which occurred on April 7, 2014, creating a global leader in the cement industry with 168 plants in 90 countries and an annual capacity of 386 million tons. The merger, achieved through a share exchange, brought financial benefits, reduce in capital costs, and significant synergies. However, the deal faced regulatory scrutiny in about

1 The source codes of the algorithms used are available on request from the author.

15 countries where asset divestments were required to preserve competition. Despite reduced earnings after the merger, the company strategically mastered these challenges and consolidated its position in the market. For the Lafarge-Holcim merger to be successful, both companies had to obtain antitrust approvals in 17 jurisdictions, including the European Commission (Bedi and Vij, 2019).

2. Literature Review

Game theory provides a robust framework for understanding mergers and acquisitions by analysing strategic decision-making and collaboration between firms. Its application enables better prediction of transaction outcomes, improves negotiation tactics, and optimizes business strategies in a dynamic market environment. The application of game theory in economics has a long history, beginning with van Neumann and Morgenstern's pioneering work "Game Theory and Economic Behavior" in 1944. Subsequent contributions from Nash (1951) on non-cooperative games and Shapley (1953) on cooperative games laid the foundations for modern economic analysis. In professional literature, several indices are used to evaluate the decision-making power of players. The most well-known are the Shapley – Shubik index, the Banzhaf index, the Johnston index, the Holler index, and the Deegan Packel index. In this article, we will focus on two of these indicators – the Shapley – Shubik index and the Banzhaf index, which will be used to calculate the decision-making power of individual subjects.

The Banzhaf index evaluates a member's voting power or influence within a coalition by assessing the criticality of their participation in achieving the coalition's goals (Banzhaf, 1965). The Shapley – Shubik index expands on these principles by analyzing the likelihood that a coalition member will be key in decision-making processes. The primary difference between the two lies in the theoretical characteristics they meet, which makes one more suitable than the other, depending on the nature of the problem at hand (Shapley and Shubik, 1954) (Feltkamp, 1995).

De Keijzer et al. They expanded on these principles by examining the problem of the inverse power index, which determines how coalition weights can be adjusted to achieve a specific distribution of power. By modelling scenarios, companies can anticipate competitors' actions, evaluate potential risks and benefits, and develop optimal strategies. Such analyses and predictions of behaviour in a competitive environment are particularly valuable for SMEs seeking to build coalitions that are effective and fair. They provide businesses with useful tools to propose cooperation strategies that are in line

with Antimonopoly Office of the Slovak Republic (AMO SR) guidelines and ensure that coalitions bring economic and legal benefits. It deals with the inverse problem of the Banzhaf index, from the point of view of determining the distribution of votes or weights among players to generate a specific distributed decision-making power. This problem is very relevant for the creation of cooperative structures in the business environment – if companies want even or proportionally balanced decision-making, it is necessary to calculate the corresponding weights. Edwards and Weichenrieder note that there are an infinite number of such weighing systems that lead to the same decision-making power, which opens up the possibility of optimization according to other criteria (transaction costs, fairness, efficiency) (Keijzer, et al., 2010) (Edwards and Weichenrieder, 2009).

The main drivers of M&A include access to resources, reduced operating costs and increased market presence. However, achieving these benefits requires careful alignment of objectives, trust among members, and adherence to regulatory standards. Holubčík and Soviar identified mutual trust, compliance with contractual obligations and clearly defined benefits as critical factors for successful cooperation. These findings underline the importance of effective management of cooperation, particularly in ensuring that partnerships comply with the law (Beck, et al., 2005) (Holubčík and Soviar, 2021).

Regulatory oversight can foster innovation and competitiveness by providing a framework for permissible cooperation. For example, Nagarajan and Sošić have demonstrated that coalition stability can be maintained through far-sighted agreements in which members weigh the long-term benefits of cooperation rather than focusing only on immediate gains. This approach to align with the AMO guidelines, which emphasize the importance of transparency and consumer welfare in cooperation agreements (Nagarajan and Sošić, 2007).

Traditionally, a player's decision-making power in a cooperative game is expressed through the Banzhaf or Shapley – Shubik index. However, these indicators usually ignore the specific context in which the player (company) operates; therefore, they propose a reinterpretation of the Banzhaf index in a qualitative comparative analysis (QCA) environment, where each player represents a condition and the "coalitions" correspond to different combinations of conditions leading to the desired outcome. This approach makes it possible to capture better the differential impact of individual elements in complex configurations, which is particularly important when analysing the behaviour of small and medium-sized enterprises in a regulated competitive environment (Haake and Schneider, 2024).

Álvarez-Mozos and Tajeda propose an extension of Banzhaf-Owen for games with a structured level of cooperation, which they refer to as "Banzhaf level value." This can be used in many electoral situations (Álvarez-Mozos and Tejada, 2011).

2.1 Regulatory foundations: Balance between cooperation and competition

Competition law plays a significant role in regulating cooperation efforts between undertakings and ensures that such agreements do not distort market dynamics. In the European Union, competition is mainly governed by Articles 101 and 102 of the Treaty on the Functioning of the European Union (TFEU). While Article 101 prohibits agreements that could restrict competition, such as price fixing, market sharing or production restrictions, it allows for exceptions for collaborations that bring consumer and technological benefits, which must outweigh their potential restrictive effects (European Union, 2016).

The development of antitrust policy shows a shift from legal formalism to economic analysis, allowing for a more accurate assessment of competitive practices**.** Kovacic and Shapiro point to the increasing use of empirical methods and game theory to help distinguish beneficial cooperation from forms of coordination that can harm competition. Motta (2004) emphasizes that properly set regulations can promote competition if they allow cooperation in areas with high barriers to entry, such as research and development. On the contrary, Nagarajan and Sošić warn that unclear regulation may lead to a reduction in strategic alliances, which will reduce the willingness of companies to invest in joint projects (Kovacic and Shapiro, 2000) (Nagarajan and Sošić, 2007).

In Slovakia, competition supervision is carried out by the Antimonopoly Office of the Slovak Republic (AMO SR), which issues guidelines to distinguish between permissible forms of cooperation and practices that may distort competition. Acceptable forms include joint purchasing, sharing of resources, or market development initiatives, as long as they lead to efficiency gains and cost reductions without negatively impacting the competitive environment. Such frameworks are particularly important for SMEs, which often lack the resources to challenge dominant market players independently. Its main task is to control and regulate the competitive environment in the market, focusing on eliminating cartel agreements, monitoring mergers and acquisitions, and preventing abuse of dominant positions.

Act No. 187/2021 Coll. on the Protection of Competition regulates the conditions under which companies can cooperate without restricting competition. For agreements be-

tween competitors (so-called horizontal agreements), the maximum market share is set at 10%. For vertical agreements (between undertakings at different levels of the supply chain), the limit is 15%. If these limits are not exceeded, the agreement is not considered to be restrictive of competition, unless it has the object of directly distorting the competitive environment (Act No. 187/2021 Coll. on the Protection of Economic Competition and on Amendments and Supplements to Certain Acts of the National Council of the Slovak Republic, dátum neznámy).

Business-to-business cooperation, such as joint purchasing agreements, resource sharing, or market support initiatives, may be allowed if it has a proven track record of efficiency gains, cost savings, and benefits for consumers, while not distorting competition. The constructive collaboration effects of such partnership, particularly in the context of economies of scale and technological innovation, can be viewed as a positive factor for the market. The legal framework governing the control of mergers and joint ventures is defined in Regulation (EC) No. 139/2004. The European Commission assesses the impact of these agreements on competition and their contribution to innovation when evaluating such transactions.

These mechanisms are particularly important for small and medium-sized enterprises (SMEs), which often face limited access to resources and high barriers to market entry. Regulations, therefore, allow cooperative alliances to be formed between SMEs in order to increase their competitiveness vis-à-vis dominant market players, but within the limits of the law to avoid monopolisation.

Haake and Schneider emphasize that quantitative metrics for individual conditions can serve as a complement to the traditional indicators of compliance (coverage and consistency) in QCA. Similarly, in economic policy and regulation, the relative Banzhaf index could help the Antimonopoly Office of the Slovak Republic to understand better which companies have excessive influence within cooperative structures. In this way, regulation can be targeted more precisely, ensuring a balance between promoting cooperation and protecting competition (Haake and Schneider, 2024).

Regulatory policies thus underline that properly structured cooperation between SMEs can strengthen market competition rather than limit it. By developing common strategies and sharing expertise, SMEs can reduce the market concentration of large enterprises, while promoting innovation potential and market diversification. Legal restrictions ensure that cooperation does not lead to unfair practices, cartels, or monopolistic behaviour.

3. Strength indicators in cooperative game theory

the modelling of such conflicting decision-making situations, where players have the opportunity to conclude binding coalition groupings, is explored by the theory of cooperative games. The basic indicators of estimating a player's strength are the Banzhaf Index (BI) and the Shapley – Shubik Index (SSI) (Shapley and Shubik, 1954) (Banzhaf, 1965). Since the analysis presented in the next chapter is based on these indicators, in this section we formalize the basic relationships of their calculation. Let's consider the number of players n and let P be a set of players, $P = \{1, 2 \dots n\}$. Let the coalition S be each subset $S \subseteq P$. The real function, defined for all subsets $S \subseteq P$, that assigns each coalition its payment, is called the characteristic function of cooperative play and has the following properties (Dlouhý and Fiala, 2015):

- $v(\emptyset) = 0,$ (1)
- $v(S_1 \cup S_2) \geq v(S_1) + v(S_2),$ (2)

for all disjoint pairs of coalitions $S_1\ S_2 \subseteq P$. We refer to the pair (P, v) as a cooperative game. The characteristic function assigns to each coalition $S \subseteq P$ value $v(S)$ that represents its "strength" and is referred to as the value of the coalition. The relation (2) expresses the superaditivity of the characteristic function, which means that the union of two disjoint coalitions cannot produce a coalition with a profit less than the sum of the profits of these coalitions acting separately. If a characteristic function assigns only the values 1 or 0 to each coalition $S \subseteq P$, we speak of a simple game. If $v(S) = 1$, the coalition is victorious, if $v(S) = 0$, it is a non-winning coalition. A simple game is also referred to as a voting game (Dlouhý and Fiala, 2015)

Several concepts have been developed to solve games in the characteristic function form, but none of them guarantees a straightforward solution to a given game. However, various approaches have been proposed to evaluate a player's position in a specific conflict. In 1953, the American mathematician and economist Lloyd S. Shapley proposed a method for estimating a player's strength based on his marginal contribution to all coalitions of which he could be a member (Shapley, 1953). In 1954, Lloyd Shapley and Martin Shubik published an article proposing a modified form of Shapley's value for use in simple voting games to measure electoral power (Shapley and Shubik, 1954). This approach later became known as the Shapley – Shubik power indicator. It is based on the principle that players enter a coalition in a random order, with each such order having the same probability of occurrence. The index tracks at what point a player becomes the decisive

member of the coalition, i.e., the one who is the first to ensure that the coalition achieves the necessary majority.

Applications of simple games (with a characteristic function $v(S) = 1$ in the case of a winning coalition and $v(S) = 0$ otherwise) can be found mainly in analyses of political (voting) games. Let us assume that each i-th player from the set $P = \{1,2, \ldots n\}$ is assigned with the value $\alpha_i \in P$. This is usually the number of deputies of i-th political party, but in general, this value can be considered as the evaluation (payout) of the player in the given game. Next, let:

$$a_0 = \sum_{i=1}^{n} a_i \tag{3}$$

The constant α denotes the critical value for the victory of the coalition. For the winning coalition S, the following applies in the case of voting games:

$$\sum_{i \in S} a_i - int(\alpha a_0) > 0 \tag{4}$$

If the relation (4) holds for the coalition S than $v(S) = 1$, then, otherwise $v(S) = 0$.

The Shapley – Shubik index assigns each player $i \in S$ a rating according to their relationship h_i :

$$h_i = \sum_{i \in S} \left\{ \frac{(|S| - 1)!\,(n - |S|)!}{n!} \right\} \tag{5}$$

where the summation takes place through all majority coalitions S $v(S) = 1$, so that $v(S - \{i\}) = 0$ is a minority coalition. The following applies to index values: $\sum_{i \in S} h_i$ $1; h_i \geq 0, i \in S$.

The Shapley – Shubik index can be interpreted as the probability that the i-th player is essential in the formation of all majority coalitions. This index is often referred to as the coalition potential of the player (Dlouhý and Fiala, 2015).

A new measure of distribution of voting power was introduced in 1965 by John F. Banzhaf. The Banzhaf index is used to express the relative decision-making power of a player in a voting game, based on his participation in the so-called critical coalitions. A critical coalition is understood as a coalition in which the absence of a specific player would cause the coalition to lose its majority support. In such a case, the player has a decisive influence on whether the coalition will be victorious or not. The number of such cases enters into the calculation of the Banzhaf index, which is as follows (Banzhaf, 1965):

$$\beta_i = \frac{e_i}{\sum_{i \in S} e_i}, i \in S \tag{6}$$

Where e_i – the number of coalitions in which the *i*-th player is decisive. The summation in the denominator occurs across all majority coalitions S (v(S) =1) such that $v(S - \{i\}) = 0$ a is a minority coalition. The following applies to index values: $\sum_{i \in S} \beta_i = 1; \beta_i \geq 0, i \in S$.

The classic interpretation of the above indices is based on the fact that the strength of the player is directly proportional to the value of the index, but in the following analysis we will focus on the reverse interpretation, where it is desirable for players to achieve the lowest possible value of these indices.

We will introduce the terms Reverse Banzhaf Index (RBI) and Reverse Shapley – Shubik Index (RSSI). The proposal of the Reverse Banzhaf Index (RBI) and Reverse Shapley – Shubik Index (RSSI) represents our original methodological contribution, as it adapts traditional power indices to the regulatory environment of antitrust law, providing a novel tool for assessing coalition potential in highly concentrated markets. The indices will be calculated for enterprises operating in the Health Laboratory Services sector in the Slovak Republic (elements of the *P* set), while the values $a_i, i \in P$ represent their market shares. We will examine the possibilities of concluding horizontal agreements and thus set the critical value α to 0.1 (10%), which is a statutory condition for the protection of competition.

The logic behind the reverse interpretation of indices is that if a coalition exceeds the critical value of α (percentage of market share), it is considered inadmissible. Therefore, to set the values of the characteristic function, we will use a modified relation:

$$\sum_{i \in S} a_i - \alpha a_0 > 0 \tag{7}$$

If the relation (7) is true, then v(S) =1 and v(S) =0otherwise. Coalitions S where v(S) =1 we interpret as inadmissible and coalitions where v(S) =0 that we interpret as admissible. That is, if the new coalition falls apart by adding a company, the value of the indicators will increase. From the point of view of this index, the company with the lowest value of indicators has the greatest strength, as it has the most opportunities to join coalitions and thus strengthen their market power.

It should be emphasized that the reverse Banzhaf index used in this work is not the same as the inverse problem known from the literature on weighted voting games. While the reverse index represents an alternative measure of a player's influence in settings with

an upper limit (e.g., market share), the inverse problem concerns the design of a voting game that best reproduces a predetermined distribution of power among players.

4. Analysis of the coalition strength of small and medium-sized enterprises of the specific sector of the slovak republic

In this section, we will apply the reverse Banzhaf and Shapley – Shubik index to a sample of enterprises operating in the sector 86901 - Medical laboratory services in the Slovak Republic. The aim is to quantify the relative weight of individual entities in cooperation scenarios, especially in the context of potential mergers, alliances, or joint business activities. We will consider the key parameters to be market share, the amount of annual sales, and the amount of profit. These variables represent the economic strength of each entity and its contribution to the potential collective value of the coalition. The data sources are the financial statements of nine companies for 2022 and market share data published by the Slovak Investment and Trade Development Agency in 2024 (SARIO, 2024).

Given that the existing competition rules directly influence the results of the indices, we base their application on the legislative frameworks of the European Union and the Slovak Republic, as presented in detail in the introduction. It sets critical thresholds for the admissibility of coalitions, particularly the 10% market share limit for horizontal agreements. These rules form the basis for defining permissible and inadmissible cooperative configurations between firms in the sector.

The source of data is publicly available economic statements of companies, company databases, and expert studies focused on the development of the market environment. In view of regulatory constraints, it is necessary to respect a maximum cumulative market share of 10% for horizontal agreements and 15% for vertical agreements when building coalitions (Act No. 187/2021 Coll. on the Protection of Economic Competition and on Amendments and Supplements to Certain Acts of the National Council of the Slovak Republic, dátum neznámy).

We will focus exclusively on horizontal coalitions, i.e., between companies operating at the same level of the market (e.g., providers of the same type of laboratory services). This approach corresponds to the legislative framework, which sets a stricter limit of 10% of market share ($\alpha = 10\%$) for horizontal agreements.

To ensure consistency and comparability of data, we assume that corporate revenues reported in the financial statements are derived exclusively from sales of products

in the sector. We interpret the EBITDA indicator as an indicator of operational performance, which reflects only operating income and costs, excluding the impact of extraordinary items or capital structure. We also assume homogeneity of operating costs, such as energy costs, rental, or investments in research and development, because with a larger volume of production, unit costs are reduced. Those assumptions should enable a fair comparison of undertakings, regardless of their size. The analysed sample includes nine separately identified companies that met the criteria in terms of the availability of data on sales, profit, and market share. The category **'other'** comprises around 20 to 25 smaller entities with an individual market share of less than 1%, for which sufficiently reliable data were not available for a separate evaluation. Their combined market share is 15.4%, and they are included only for the purpose of quantifying the overall market concentration. These entities are no longer included in the calculation of indices. The aggregate data of the analysed enterprises, which form the basis for further quantitative evaluation and interpretation of the results, are presented in Table 1.

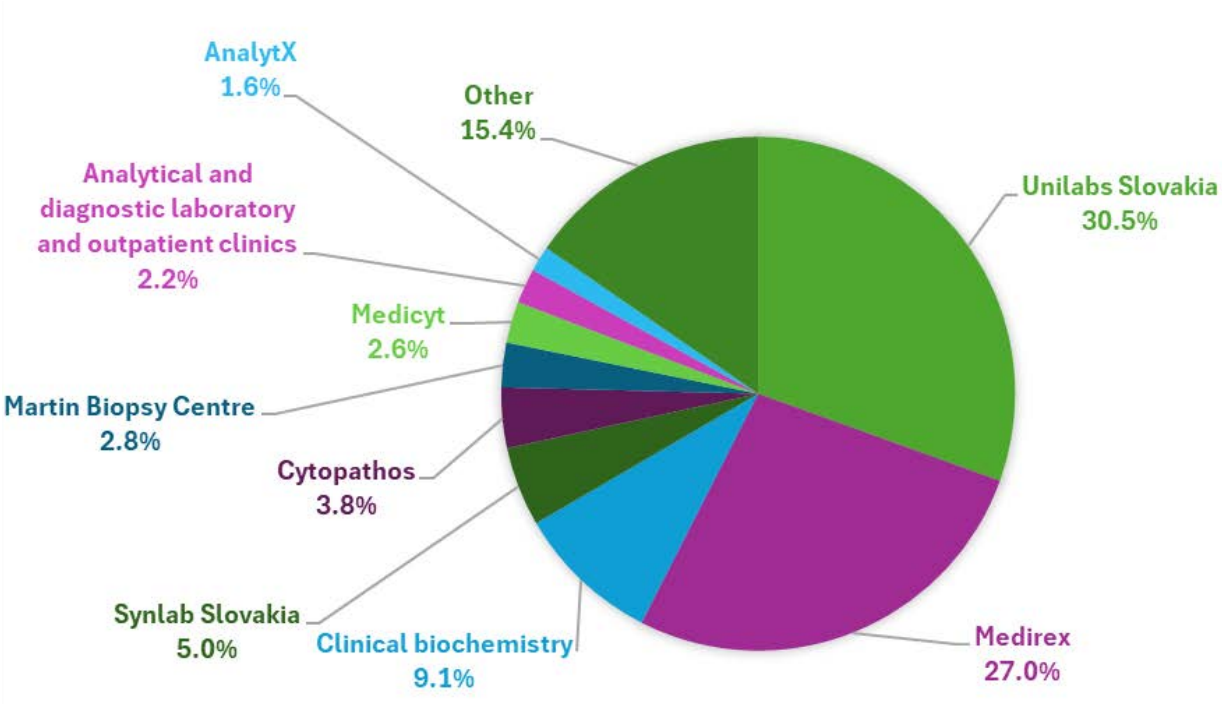
Table 1: Industry input data for 2022

Company	Market share (%)	Total sales 2022 (€)	Total profit 2022 (€)
Unilabs Slovakia	30.5%	97,226,549 €	841,764 €
Medirex	27.0%	85,829,375 €	17,528,146 €
Clinical Biochemistry	9.1%	25,174,656 €	9,887,609 €
Synlab Slovakia	5.0%	18,307,389 €	1,777,318 €
Cytopathos	3.8%	11,882,253 €	924,946 €
Martin Biopsy Centre	2.8%	9,143,668 €	150,526 €
Medicyt	2.6%	8,412,792 €	1,916,185 €
Analytical-diagnostic laboratory and outpatient clinics	2.2%	6,553,355 €	1,102,224 €
AnalytX	1.6%	5,399,516 €	998,755 €
Other	15.4%		

Source: SARIO, NUE, own processing

The market shares of the companies are shown in Figure 1.

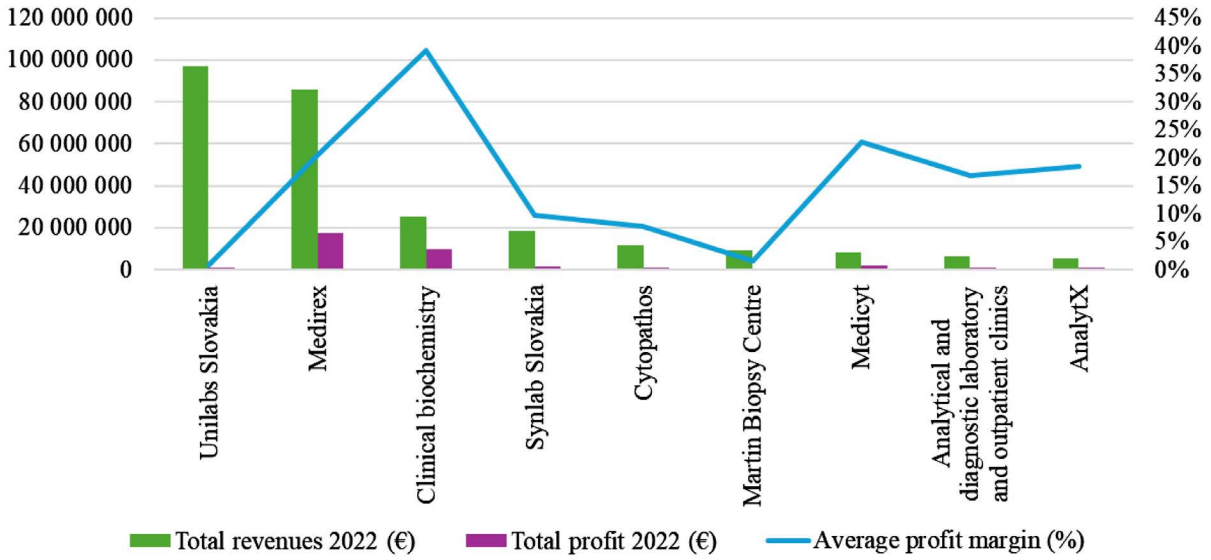
Figure 1: Market shares in 2022



Source: own processing

The dominant players in the industry are *Unilabs Slovakia* (30.5%) and *Medirex* (27%). This points to a high concentration of the market, where the two largest companies control more than half of the sector. On the other hand, smaller entities, such as *Klinicka biochémia* (9.1%), *Synlab Slovakia* (5.0%), *Cytopathos* (3.8%), and *Medicyt* (2.6%), have significantly lower shares, which indicates fragmentation of the rest of the market. Figure 2 shows: total sales, profit (EBITDA), and profit-to-sales ratio (PZM) as a measure of operational efficiency. The green bars represent the volume of revenues, the purple ones represent the absolute level of profit, and the blue line shows the ratio of EBITDA to sales in percentage terms, providing a more comprehensive overview of the economic performance of individual companies in 2022.

Figure 2: Financial indicators of companies for 2022



Source: own processing

It follows from the above that although Unilabs Slovakia and Medirex achieve the highest sales, which correlates with their market size, taking into account profit margins, it turns out that high turnover is not always synonymous with high profitability. An example is Medicyt, which, despite a relatively low market share (2.6%), shows a high level of profitability, which signals effective cost management and competitiveness potential. This discrepancy between size and efficiency can provide a basis for a more in-depth analysis of the competitive advantages and strategic positioning of individual entities.

Next, we will focus on the identification of permissible coalitions, i.e., coalitions with a total market share of less than the critical value of $\alpha = 10\%$ (their $v(S) = 0$). On the contrary, coalitions whose joint share exceeds this limit were evaluated as inadmissible (their $v(S) = 1$). In this way, we identify companies that are most often found in permissible cooperative combinations and therefore have the greatest room for creating effective alliances without violating competition rules. The results are summarized in Table 2, which shows the frequency of occurrence of individual companies in permissible cooperation groupings.

Table 2: Abundance in permissible coalitions

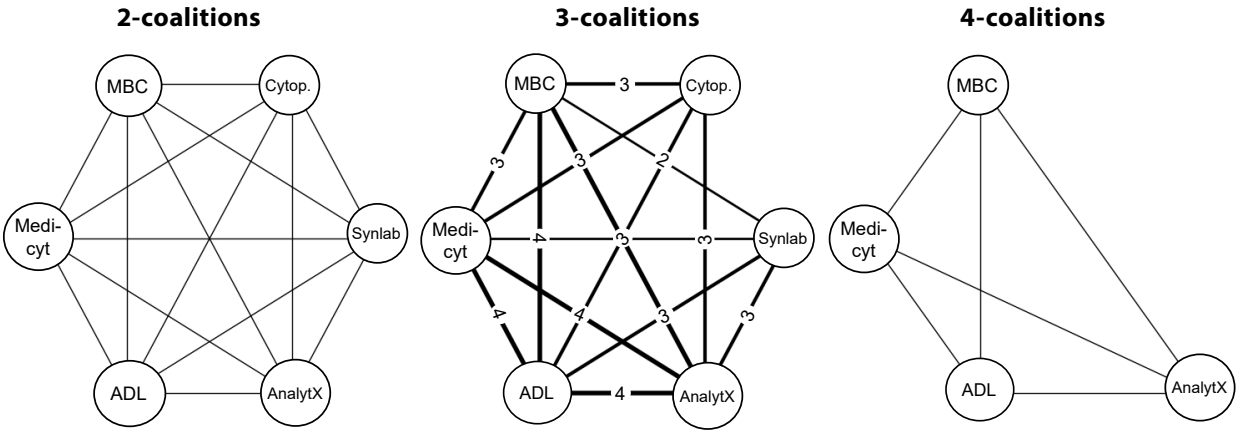
Firm	Number of occurrences	Market share (%)
Unilabs Slovakia	1	30.5%
Medirex	1	27.0%
Clinical Biochemistry	1	9.1%
Synlab Slovakia	10	5.0%
Cytopathos	12	3.8%
Martin Biopsy Centre	14	2.8%
Medicyt	15	2.6%
Analytical-diagnostic laboratory and outpatient clinics	15	2.2%
AnalytX	16	1.6%

Source: own processing

The results clearly show that larger companies with a dominant market position have significantly limited opportunities to enter into coalitions, which is a consequence of the risk of exceeding the regulatory limits of market share. For example, Unilabs Slovakia and Medirex, as the two largest players, appear in only one permissible coalition. By contrast, smaller businesses such as AnalytX (1.6%) or ADLs and ambulances (2.2%) show the highest number of occurrences (16 and 15, respectively), confirming greater flexibility and better adaptability within strategic alliances. Notably, despite a slight difference in market share (0.4%), ADL, ambulances, and Medicyt agree on the number of occurrences in permissible coalitions. This fact shows that market share is not the only determining factor of coalition strength – the company's ability to "fit" into cooperative configurations without exceeding legislative thresholds is also essential. Such a quantitative view of cooperation brings a new perspective on the formation of alliances in the regulated sector and can be the basis for further research in the field of strategic decision-making in oligopolistic structures.

To complement the numerical results in Table 2, we also constructed a network visualisation of admissible coalitions. Figure 3 shows the network of 2, 3 and 4 member coalitions, where the nodes represent individual laboratories and the edges corresponds to the number of admissible coalitions in which a given pair of firms co-occurs.

Figure 3: Network representation of admissible coalitions



Source: own processing

Figure 3 visualizes the network of admissible coalitions among the six laboratories. In the 2-coalition panel, edges between firms represent all admissible pairs whose combined market share does not exceed the 10% threshold. In the 3-coalition panel, the edges show how many times a given pair of firms appears together in an admissible three-firm coalition, while the 4-coalition panel displays the only admissible four-firm coalition.

From this perspective, it is immediately visible that smaller laboratories are embedded in a dense web of potential alliances, while dominant players are connected only by a few permissible links. The visualisation thus confirms that SMEs have much greater room to form coalitions without breaching the 10% market-share threshold and supports the interpretation of the results discussed above. Next, we will focus on the quantification of the Reverse Banzhaf Index (RBI), which can be a tool for quantitative evaluation of the negotiation flexibility of companies in coalition structures within a regulated market. A higher value of this index signals that the participation of a given entity in an otherwise permissible coalition often leads to its transformation into an inadmissible one, as a result of exceeding the market share limit set by legislation. The admissibility of coalitions is determined according to the characteristic function defined by relations (6) and (7), in which only those groupings whose combined market share does not exceed the defined critical value of α are considered admissible.

The calculation of the RBI was based on the identification of all permissible coalitions and subsequent testing of the number of cases in which the connection of the analyzed entity would cause a violation of the condition of admissibility according to the relationships

(6) and (7). Each such case was recorded as a contribution to the index value of the respective firm. The resulting values thus reflect the degree of regulatory limitation in the formation of strategic alliances, while a lower index value indicates a higher degree of coalition adaptability.

The values of the RBI for the analysed subjects are shown in Table 3.

Table 3: Reverse Banzhaf Index

Firm:	Reverse Banzhaf Index
Unilabs Slovakia	0.2
Medirex	0.2
Clinical Biochemistry	0.2
Synlab Slovakia	0.105263
Cytopathos	0.084211
Martin Biopsy Centre	0.063158
Medicyt	0.052632
Analytical-diagnostic laboratory and outpatient clinics	0.052632
AnalytX	0.042105

Source: own processing

The highest RBI values are achieved by Unilabs Slovakia, Medirex, and Klinicka biochémia, indicating their limited space for strategic connections with other entities. Particularly interesting is the case of Clinical Biochemistry, which, despite a lower market share (9.1%), shows the same index as the two dominant players. This may be a consequence of the company's specific position in market configurations, where even a slight increase in the aggregate share leads to a breach of regulatory limits. On the other end of the spectrum, there are companies with low RBIs, such as AnalytX, ADL, as well as Ambulances, and Medicyt, which exhibit high coalition flexibility. These companies can enter a wider range of alliances without jeopardizing the admissibility of the coalition. Interestingly, Medicyt (2.6% TP) and ADL and ambulances (2.2% TP) exhibit the same RBI of 10, despite their different market shares. This fact shows that the decisive factor for

a negotiating position is not only the absolute share, but also the structure and the number of permissible coalition combinations in which the company can participate. In some cases, entities with a higher share may face similar coalition restrictions as smaller companies. In the middle band, companies such as Synlab Slovakia and Cytopathos, exhibit medium-high RBI values. Although their involvement in coalitions is more frequent than that of dominant players, it is also limited by the fact that even a relatively slight increase in their market share in combination with other companies can lead to exceeding the set regulatory thresholds.

As a complementary analytical tool, we will also present the Reverse Shapley – Shubik Index (RSSI) quantified based on the relationships (5) and (7), which expresses the probability that the presence of a particular entity will cause the transformation of a permissible coalition into an inadmissible one due to exceeding the market limit. This indicator thus quantifies the extent to which the company is a limiting element within strategic alliances. A higher RSSI value indicates that the entity is more likely to cause the coalition to be inadmissible, which makes its negotiating flexibility low. On the contrary, low values indicate that the company is adaptable to the coalition and can participate in several cooperation groupings without incurring significant regulatory risk. The results are summarized in Table 4.

Table 4: Reverse Shapley – Shubik Index

Firm	Reverse Shapley – Shubik Index	Market share (%)
Unilabs Slovakia	0.297222	30.5%
Medirex	0.297222	27.0%
Clinical Biochemistry	0.172222	9.1%
Synlab Slovakia	0.059127	5.0%
Cytopathos	0.047222	3.8%
Martin Biopsy Centre	0.037698	2.8%
Medicyt	0.031746	2.6%
Analytical-diagnostic laboratory and outpatient clinics	0.031746	2.2%
AnalytX	0.025794	1.6%

Source: own processing

The highest RSSI values were achieved by Unilabs Slovakia and Medirex (0.297), i.e., entities with the largest market share. This result reflects their significant limitation by regulatory limits, as their entry into the coalition most often leads to its inadmissibility. In other words, there are a high number of coalitions that would otherwise be permissible. Still, when one of these companies is added, they exceed the established market share limit and become inadmissible. For these companies, this means significantly reduced room for maneuverer in forming alliances, which is desirable from the point of view of competition protection but limiting from the point of view of strategic decision-making. Also interesting is the result of Clinical Biochemistry (0.172), which, despite a lower market share, shows a significant limitation in coalition possibilities. This phenomenon can be explained by the fact that a company often finds itself in a position where its presence in otherwise permissible coalitions causes the regulatory threshold of α to be exceeded. In other words, the entry of Clinical Biochemistry into the coalition often leads to its reclassification as inadmissible, indicating that not only the size of the share is decisive, but also the combinatorial relationships with other entities in the sector.

The lowest index values are reported by entities such as AnalytX (0.026), ADL and ambulances (0.032), and Medicyt (0.032). These companies are strategically flexible, and their involvement in the coalition rarely disrupts the competitive conditions. In practice, this means that they have a high potential for concluding horizontal partnerships, which can strengthen their bargaining position vis-à-vis larger competitors. Although they do not have a significant market share on their own, thanks to their low regulatory burden, they can dynamically participate in cooperation groupings and thus effectively strengthen their competitiveness.

Together, the RBI analysis and the RSSI confirm that formal market size may not be the only factor in bargaining power. True opportunities to enter into alliances are shaped by the complex interaction of regulatory frameworks, market importance, and the firm's structural position in the industry. For businesses, this means considering not only internal efficiency but also external regulatory barriers in their decision-making.

These insights are crucial for corporate strategies, as firms with higher RBI and RSSI must be more accountable for regulatory hurdles in their decisions. In contrast, firms with lower RBI values could actively form market coalitions and use their flexibility to maximize competitive advantage.

They also provide valuable information for regulators, who can more accurately identify which entities have a real ability to influence market equilibrium based on this analysis.

At the same time, such a distribution of reversal indices reveals an asymmetric distribution of the regulatory burden – dominant players are heavily bound by caps, while smaller entities benefit from greater decision-making freedom. This regulatory balance paradox reveals that while potential distortions of competition prevent large firms from expanding, smaller firms can react more agilely and strategically manoeuvre within permissible configurations. For economic policymakers, it is a practical tool for identifying market-influencing entities whose actual impact extends beyond formal market shares.

Conclusion

Strategic cooperation between businesses, if properly regulated and optimised, can give SMEs a competitive advantage over large market players. From game theory, indices are known that make it possible to quantify the strength of individual players and their coalition potential. In this article, we used the Banzhaf and Shapley – Shubik indices, which are used by default in coalition games. We focused on evaluating the coalition potential of companies from a specific sector that can be characterized as small and medium-sized enterprises. Regarding the regulatory legislative specifics of coalition formation, we have introduced a reverse interpretation of these indices. The presented procedure can help smaller companies better estimate their own coalition potential, as well as the potential of their competitors and potential coalition partners.

In general, cooperation can lead to cost reductions, sharing of resources, and sharing of know-how, which can strengthen SMEs' position in the market. This can promote competition and bring benefits not only to businesses but also to end consumers.

Vertical agreements between operators at various levels of the distribution chain, such as those between a provider and a customer of laboratory services, were not the subject of this analysis, as no such links were identified in the publicly available data for the sector under review. Extending the analysis to vertical structures would require a different methodological framework and may be the subject of separate research in the future.

Given the dynamics of today's markets and the ever-evolving regulatory frameworks, it is essential for businesses to constantly evaluate and adapt their strategic alliances to maximize their benefits while minimizing the risks associated with market concentration.

We see suggestions for further expansion of the presented approach, especially in the area of dynamic modelling of the development of coalition structures over time, extension of the analysis to multi-year data, and testing indices in other regulated sectors, where small and medium-sized enterprises face similar competitive challenges. The presented indices could directly serve as a basis for the redistribution of the surplus from cooperation, where such surplus should be redistributed among the members of the formed coalition inversely proportional to their index values. It is also clear that some coalitions may be more successful than others. A promising area is undoubtedly the analysis of the possibilities for quantifying the constructive collaboration effect of potential coalitions, thereby providing a more realistic estimate of a player's position in a given game. In such a case, it would be possible to provide an invaluable tool to regulators, enabling them to identify critical players.

This article extends the methodology for evaluating the cooperative behaviour of SMEs in a regulated market environment by introducing a reverse interpretation of decision-making power indicators. Indices defined in this way enable more accurate identification of entities that have a real potential to form permissible strategic alliances without distorting competition rules. The analytical approach presented can serve as support for the decision-making of smaller companies, as well as a tool for regulators to monitor market concentration. Future research could follow up by extending the model to include dynamic elements, quantifying constructive collaboration effects in coalitions, or applying it to other sectors with increased levels of regulation. Long-term monitoring of the impacts of coalition structures on competitiveness and market balance has the potential to bring knowledge not only for business practice, but also for the creation of economic policy with a direct social impact.

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