

# A NONLINEAR SUPPLY-DRIVEN INPUT-OUTPUT MODEL

Nooraddin Sharify\*

## Abstract

One of the major limitations of the supply-driven input-output (I-O) Ghosh model concerns its linear production function. Using the I-O table, this paper replaces the linear production function with the Cobb-Douglas (CD) production function within the supply-driven model. The two models are compared both theoretically and empirically. Nonlinear production function, relative substitutability of primary factors, and variability of the proportion of intermediate inputs over product levels are the characteristics of the proposed model. The consideration of sectors' Solow residual as Total Factor Productivity (TFP) of sectors is yet another characteristics of the proposed model. The model is also plausible in value added and supply shock computations.

**Keywords:** nonlinear input-output model, Cobb-Douglas production function, Ghosh model, plausible supply-driven input-output model

**JEL Classification:** C63, C67, E27

## 1. Introduction

I-O models are mainly based upon linear functions. The Ghosh supply-driven I-O model is one of these models, which has a linear function form. Although the linear functions are adequate for a number of empirical cases, they have some characteristics that prevent them from being used for many purposes.

There are many conditions, in which variables may have nonlinear relationships with each other. In addition, there are some other problems that make the Ghosh supply-driven I-O model implausible. The perfect substitution of primary inputs is one of these, which assumes that these inputs can be substituted by each other perfectly. The assumption of perfect complementarity of aggregate primary inputs with intermediate inputs is taken into account as another problem of this model. Such characteristics lead this model to be considered as implausible.

One attempt has been made to propose a nonlinear I-O model by Zhao *et al.* (2006). Although the model is a great contribution to the non-linear I-O model, it fails to compute the parameters of the model from the I-O table. Besides, the estimation of the model is, to some extent, dependent upon the planner's decisions.

To overcome these deficiencies, this paper suggests a CD production function instead of a linear one. The parameters of the model are specified from an I-O table. The model has a number of characteristics, which seem more suitable if employed in empirical cases.

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\* Nooraddin Sharify, Department of Economics, University of Mazandaran, Babolsar, Iran (nsharify@umz.ac.ir).

The components of primary inputs can be relatively substituted for each other. The intermediate and primary inputs can also be substituted relatively in the proposed model. Hence, the implausibility problem of the Ghosh supply-driven model will be removed. In addition, the model allows the researchers to specify the TFP of sectors through Solow residual. Finally, all parameters of the model can be calculated from the I-O table.

The paper contains four sections. The Ghosh model is reviewed in the second section. Capabilities and deficiencies of the model are also reviewed in this section. The third section develops the proposed model. In addition, the characteristics of the proposed model are demonstrated in this section. The implementation of the proposed model has been presented in the fourth section. Results of the proposed model are compared with those of the Ghosh model. And finally, the concluding section will end the paper.

## 2. The Ghosh Model

This model proposed by Ghosh (1958) is in value terms. The model was proposed to relate gross outputs of industries to supply factors. It relies on the constancy of  $b_{ij}$ , the direct coefficient of consumption of products for different levels of outputs:

$$b_{ij} = \frac{x_{ij}}{q_i} \Rightarrow x_{ij} = b_{ij} \cdot q_i, \quad (1)$$

where  $x_{ij}$  refers to transaction between sector  $i$  to  $j$ , and  $q_i$  refers to total output of sector  $i$ . Hence, the total inputs of sectors can be formulated as follows:

$$\mathbf{q} = \mathbf{qB} + \mathbf{v} \Rightarrow \mathbf{q} - \mathbf{qB} = \mathbf{v} \Rightarrow \mathbf{q} = \mathbf{v} \times (\mathbf{I} - \mathbf{B})^{-1} = \mathbf{v} \times \mathbf{G}, \quad (2)$$

$\mathbf{B} = [b_{ij}]$  denotes the matrix of the fixed intermediate output coefficient,  $\mathbf{q}$  the row vector of total inputs of sectors,  $\mathbf{v}$  the row vector of primary inputs of sectors, and  $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$  the matrix of Ghoshian inverse.

This model has some characteristics and is employed for different purposes. Bon (1988), Dietzenbacher (1989, 1997) and Lenzen *et al.* (2010) refer to the Ghosh supply driven I-O model to be as plausible as the Leontief demand driven I-O model. It has been employed for forward linkage in some studies such as Augustinovic (1970) and Cai (2004). Dietzenbacher (1997) advocates using this model for price change analysis. In addition, a modified version of this model was also implemented by Gallego and Lenzen (2005), which divides responsibility into mutually exclusive and collectively exhaustive portions, that are assigned to producers and consumers as shared responsibilities.

However, since the constant amount of  $g_{ij}$  shows the effect of a unit change in primary inputs of sector  $j$  on the level of total products of sector  $i$ , it is concluded that: 1) Because of the products of sectors they have linear relationships with respect to all primary factors, the marginal impact of primary inputs on total products of sectors is constant for different levels of these inputs. As a result of this characteristics, the aggregate primary and intermediate inputs are perfectly complementary. 2) The effect of a unit of different

kinds of primary inputs on products of sectors is the same. Hence, a unit of labour force and capital have the same effect on the level of products of sectors. On the other hand, these inputs are perfectly substitutable for each other, so it is possible to release one of these inputs in the production process.

Note that, due to perfect complementing relationship between total intermediate and total primary inputs of sectors, the implementation of the Ghosh model in impact studies suffers from a lack of credible results Oosterhaven (1988, 1989), Gruver (1989) and Mesnard (2009). To this end, Guerra and Sancho (2011) attempted to solve the implausibility problem of this model in value-added and allocation change of supply shocks. However, Oosterhaven (2012) claimed that this solution makes the model more implausible.

### 3. The Proposed Model and Its Characteristics

#### 3.1 The model

To introduce the model, let  $a_{ij}$  stands on the technical coefficient in which:

$$a_{ij} = \frac{x_{ij}}{q_j} \Rightarrow x_{ij} = a_{ij} \cdot q_j, \quad (3)$$

where  $q_j$  refers to total input of sector  $j$ .

The CD production function of Sector 1 can be developed using the related non-zero inputs and corresponding technical coefficient as the share of the inputs in production of the sector.

$$x_{11}^{a_{11}} \cdot x_{21}^{a_{21}} \cdot \dots \cdot x_{n1}^{a_{n1}} \cdot k_1^{f_1} \cdot l_1^{e_1} = m_1, \quad (4)$$

$k_1$  and  $l_1$  denote the size of capital and labour, respectively, in Sector 1.  $f_1$  and  $e_1$  denote the share of capital and labour in total inputs of Sector 1, respectively.  $m_1$  refers to the level of products of Sector 1 that is defined by the inputs.

To form a relationship between  $m_1$  and  $q_1$ , Equation 5 is introduced as follows:

$$p_1 = \frac{q_1}{m_1} \Rightarrow q_1 = m_1 \cdot p_1, \quad (5)$$

where  $p_1$ , the undefined proportion of total output of Sector 1, concerns the Solow residual of this sector.

The logarithm of Equation 5, with respect to Equation 4, allows us to change the equation into a semi-linear one.

$$\log x_{11}^{a_{11}} + \log x_{21}^{a_{21}} \cdot \dots + \log x_{n1}^{a_{n1}} + f_1 \log k_1 + e_1 \log l_1 + \log p_1 = \log q_1. \quad (6)$$

Equation 1 is employed to link production equations of the sectors. Thus, Equation 6 can be rewritten with respect to Equation 1 as follows:

$$a_{11}(\log b_{11} + \log q_1) + a_{21}(\log b_{21} + \log q_2) + \dots + a_{n1}(\log b_{n1} + \log q_n) + f_1(\log k_1) + e_1(\log l_1) + \log p_1 = \log q_1 \quad (7)$$

Now, Equation 7 is developed for all sectors:

$$\begin{cases} \log q_1 - (a_{11} \log q_1 + a_{21} \log q_2 + \dots + a_{n1} \log q_n) = a_{11} \log b_{11} + a_{21} \log b_{21} + \dots + a_{n1} \log b_{n1} + f_1 \log k_1 + e_1 \log l_1 + \log p_1 \\ \log q_2 - (a_{12} \log q_1 + a_{22} \log q_2 + \dots + a_{n2} \log q_n) = a_{12} \log b_{12} + a_{22} \log b_{22} + \dots + a_{n2} \log b_{n2} + f_2 \log k_2 + e_2 \log l_2 + \log p_2 \\ \vdots \\ \log q_n - (a_{1n} \log q_1 + a_{2n} \log q_2 + \dots + a_{nn} \log q_n) = a_{1n} \log b_{1n} + a_{2n} \log b_{2n} + \dots + a_{nn} \log b_{nn} + f_n \log k_n + e_n \log l_n + \log p_n \end{cases} \quad (8)$$

The matrix form of the model is employed to summarise the equations.

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} - \begin{bmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & & a_{n2} \\ \vdots & & & \vdots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} = \begin{bmatrix} a_{11} \log b_{11} + a_{21} \log b_{21} + \dots + a_{n1} \log b_{n1} + f_1 \log k_1 + e_1 \log l_1 + \log p_1 \\ a_{12} \log b_{12} + a_{22} \log b_{22} + \dots + a_{n2} \log b_{n2} + f_2 \log k_2 + e_2 \log l_2 + \log p_2 \\ \vdots \\ a_{1n} \log b_{1n} + a_{2n} \log b_{2n} + \dots + a_{nn} \log b_{nn} + f_n \log k_n + e_n \log l_n + \log p_n \end{bmatrix} \quad (9)$$

And then,

$$\begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & & & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix} \times \begin{bmatrix} a_{11} \log b_{11} + a_{21} \log b_{21} + \dots + a_{n1} \log b_{n1} + f_1 \log k_1 + e_1 \log l_1 + \log p_1 \\ a_{12} \log b_{12} + a_{22} \log b_{22} + \dots + a_{n2} \log b_{n2} + f_2 \log k_2 + e_2 \log l_2 + \log p_2 \\ \vdots \\ a_{1n} \log b_{1n} + a_{2n} \log b_{2n} + \dots + a_{nn} \log b_{nn} + f_n \log k_n + e_n \log l_n + \log p_n \end{bmatrix}, \quad (10)$$

where  $(\mathbf{I} - \mathbf{A}')^{-1} = [c_{ij}] = \mathbf{C}$

And finally, Equation 10 is rewritten into multiplying form to change into CD function.

$$\begin{aligned}
\begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \times \begin{bmatrix} \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \\ \log_{b_{12}}^{a_{12}} \cdot a_{22} \cdot \dots \cdot a_{n2} \cdot k_2^{f_2} \cdot l_2^{e_2} \cdot p_2 \\ \vdots \\ \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \end{bmatrix} \Rightarrow \\
\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} &= \begin{bmatrix} \left( \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \right)^{c_{11}} \left( \log_{b_{12}}^{a_{12}} \cdot a_{22} \cdot \dots \cdot a_{n2} \cdot k_2^{f_2} \cdot l_2^{e_2} \cdot p_2 \right)^{c_{12}} \cdots \left( \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \right)^{c_{1n}} \\ \left( \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \right)^{c_{21}} \left( \log_{b_{12}}^{a_{12}} \cdot a_{22} \cdot \dots \cdot a_{n2} \cdot k_2^{f_2} \cdot l_2^{e_2} \cdot p_2 \right)^{c_{22}} \cdots \left( \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \right)^{c_{2n}} \\ \vdots \\ \left( \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \right)^{c_{n1}} \left( \log_{b_{12}}^{a_{12}} \cdot a_{22} \cdot \dots \cdot a_{n2} \cdot k_2^{f_2} \cdot l_2^{e_2} \cdot p_2 \right)^{c_{n2}} \cdots \left( \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \right)^{c_{nn}} \end{bmatrix}.
\end{aligned} \tag{11}$$

### 3.2 The characteristics of the Model

One characteristics of the proposed model relies on its CD form that is the basis of relative substitutability of primary factors:

**Theorem 1:** The proposed model relies on relative substitutability of primary factors.

**Proof** Let the *Marginal Rate of Technical Substitution (MRTS)*  $K$  for  $L$  be calculated as follows:

$$\begin{aligned}
MRTS_{k \text{ for } l} &= -\frac{dk_j}{dl_j} = -\frac{\frac{\partial q_i}{\partial l_j}}{\frac{\partial q_i}{\partial k_j}} = \\
&= \frac{e_j \cdot c_{ij} \cdot l_j^{(e_j \cdot c_{ij} - 1)} \cdot k_j^{f_j \cdot c_{ij}} \left( \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \right)^{c_{i1}} \cdots \left( \log_{b_{1j}}^{a_{1j}} \cdot a_{2j} \cdot \dots \cdot a_{nj} \cdot p_j \right)^{c_{ij}} \cdots \left( \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \right)^{c_{in}}}{f_j \cdot c_{ij} \cdot k_j^{(f_j \cdot c_{ij} - 1)} \cdot l_j^{e_j \cdot c_{ij}} \left( \log_{b_{11}}^{a_{11}} \cdot a_{21} \cdot \dots \cdot a_{n1} \cdot k_1^{f_1} \cdot l_1^{e_1} \cdot p_1 \right)^{c_{i1}} \cdots \left( \log_{b_{1j}}^{a_{1j}} \cdot a_{2j} \cdot \dots \cdot a_{nj} \cdot p_j \right)^{c_{ij}} \cdots \left( \log_{b_{1n}}^{a_{1n}} \cdot a_{2n} \cdot \dots \cdot a_{nn} \cdot k_n^{f_n} \cdot l_n^{e_n} \cdot p_n \right)^{c_{in}}} \\
&= \frac{e_j \cdot k_j}{f_j \cdot l_j} = \frac{e_j}{f_j} \cdot \frac{k_j}{l_j}
\end{aligned} \tag{12}$$

Since the ratio of  $e_j/f_j$  is constant in the model, the size of *MRTS* depends on the levels of  $k_j$  and  $l_j$ . Thus, the *MRTS* of primary factors of the proposed model is related to the level of these factors. In addition, with respect to the structure of the CD production function, a perfect substitution of primary factors leads to no production in any sector. Hence, it is not possible to replace a factor with another one perfectly.

In contrast to the basic Gosh model in which the aggregate primary and intermediate inputs are perfectly complementary, it can be demonstrated that these are relatively substitutable in the proposed model:

**Theorem 2:** The aggregated primary inputs of sectors are relatively substitutable with their intermediate inputs.

**Proof** To investigate the relationship between aggregated primary inputs with total inputs and consequently with intermediate inputs of sectors, an aggregated form of value added of sectors has been replaced with wages and operation surplus. To do so,  $k_j^{f_j} \cdot l_j^{e_j}$  in Equation 11 is replaced with  $v_j^{g_j}$  in Equation 13.

$$\begin{aligned}
\begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & & c_{2n} \\ \vdots & \vdots & & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \times \begin{bmatrix} \log \frac{a_{11}}{b_{11}} \cdot \frac{a_{21}}{b_{21}} \cdots \frac{a_{n1}}{b_{n1}} \cdot v_1^{g_1} \cdot p_1 \\ \log \frac{a_{12}}{b_{12}} \cdot \frac{a_{22}}{b_{22}} \cdots \frac{a_{n2}}{b_{n2}} \cdot v_2^{g_2} \cdot p_2 \\ \vdots \\ \log \frac{a_{1n}}{b_{1n}} \cdot \frac{a_{2n}}{b_{2n}} \cdots \frac{a_{nn}}{b_{nn}} \cdot v_n^{g_n} \cdot p_n \end{bmatrix} \Rightarrow \\
\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} &= \begin{bmatrix} \left[ \left( \frac{a_{11}}{b_{11}} \cdot \frac{a_{21}}{b_{21}} \cdots \frac{a_{n1}}{b_{n1}} \cdot v_1^{g_1} \cdot p_1 \right)^{c_{11}} \left( \frac{a_{12}}{b_{12}} \cdot \frac{a_{22}}{b_{22}} \cdots \frac{a_{n2}}{b_{n2}} \cdot v_2^{g_2} \cdot p_2 \right)^{c_{12}} \cdots \left( \frac{a_{1n}}{b_{1n}} \cdot \frac{a_{2n}}{b_{2n}} \cdots \frac{a_{nn}}{b_{nn}} \cdot v_n^{g_n} \cdot p_n \right)^{c_{1n}} \right] \\ \left[ \left( \frac{a_{11}}{b_{11}} \cdot \frac{a_{21}}{b_{21}} \cdots \frac{a_{n1}}{b_{n1}} \cdot v_1^{g_1} \cdot p_1 \right)^{c_{21}} \left( \frac{a_{12}}{b_{12}} \cdot \frac{a_{22}}{b_{22}} \cdots \frac{a_{n2}}{b_{n2}} \cdot v_2^{g_2} \cdot p_2 \right)^{c_{22}} \cdots \left( \frac{a_{1n}}{b_{1n}} \cdot \frac{a_{2n}}{b_{2n}} \cdots \frac{a_{nn}}{b_{nn}} \cdot v_n^{g_n} \cdot p_n \right)^{c_{2n}} \right] \\ \vdots \\ \left[ \left( \frac{a_{11}}{b_{11}} \cdot \frac{a_{21}}{b_{21}} \cdots \frac{a_{n1}}{b_{n1}} \cdot v_1^{g_1} \cdot p_1 \right)^{c_{n1}} \left( \frac{a_{12}}{b_{12}} \cdot \frac{a_{22}}{b_{22}} \cdots \frac{a_{n2}}{b_{n2}} \cdot v_2^{g_2} \cdot p_2 \right)^{c_{n2}} \cdots \left( \frac{a_{1n}}{b_{1n}} \cdot \frac{a_{2n}}{b_{2n}} \cdots \frac{a_{nn}}{b_{nn}} \cdot v_n^{g_n} \cdot p_n \right)^{c_{nn}} \right] \end{bmatrix}, \quad (13)
\end{aligned}$$

$v_j$  refers to the value added of sector  $j$ , and  $g_j$  refers to the share of value added in total inputs of sector  $j$ .

The derivation of production function of sector  $i$  is calculated with respect to the aggregate primary factor of sector  $j$ . As it is shown in Equation 14, the effect of a unit change in the primary factor of sector  $j$  on total output of sector  $i$  is dependent on the level of the aggregated primary factor of this sector. Thus, since the production functions of sectors are non-linear with respect to primary factors, the marginal impact of these factors on total products of sectors is related to the level of these inputs.

$$\begin{aligned}
\frac{dq_i}{dv_j} &= g_j \cdot c_{ij} \cdot v_j^{c_{ij}g_j - 1} \left( \frac{a_{11}}{b_{11}} \cdot \frac{a_{21}}{b_{21}} \cdots \frac{a_{n1}}{b_{n1}} \cdot v_1^{g_1} \cdot p_1 \right)^{c_{i1}} \left( \frac{a_{12}}{b_{12}} \cdot \frac{a_{22}}{b_{22}} \cdots \frac{a_{n2}}{b_{n2}} \cdot v_2^{g_2} \cdot p_2 \right)^{c_{i2}} \cdots \\
&\quad \left( \frac{a_{1j}}{b_{1j}} \cdot \frac{a_{2j}}{b_{2j}} \cdots \frac{a_{nj}}{b_{nj}} \cdot p_j \right)^{c_{ij}} \cdots \left( \frac{a_{1n}}{b_{1n}} \cdot \frac{a_{2n}}{b_{2n}} \cdots \frac{a_{nn}}{b_{nn}} \cdot v_n^{g_n} \cdot p_n \right)^{c_{in}}
\end{aligned} \quad (14)$$

And finally, on the plausibility of the proposed model:

**Theorem 3** The proposed model is plausible.

**Proof** Since all inputs including primary factors of production sectors can relatively be substituted by others, a unit change in primary factors of a sector can influence total products of all sectors through intermediate inputs. Hence, the model is plausible to be employed for value added and supply shock computation affairs that were negotiated in previous studies.

In addition, the proposed model allows the researchers to measure the total productivity of sectors,  $p_j$ , through the Solow residual.

## 4. Results and Discussion

The empirical results of implementing the proposed and Ghosh models are examined in this section. To this end, Table 1 is employed as an example database to compare the models.

Using Table 1, Equation 15 is calculated from Equation 11. As it is shown, the products of sectors have a CD production function. The total products of sectors are dependent on the level of capital, labour and TFP of all sectors.

**Table 1 | An Example I-O Table as Database**

	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Final Demand	Imports	Total Output
<b>Sector 1</b>	240	400	0	800	500	2,000	540	3,400
<b>Sector 2</b>	0	400	600	400	600	1,800	300	3,500
<b>Sector 3</b>	800	900	700	0	800	2,200	1,000	4,400
<b>Sector 4</b>	400	700	900	800	100	2,000	500	4,400
<b>Sector 5</b>	400	600	700	800	600	2,800	900	5,000
<b>Labour</b>	560	200	500	600	1,200			
<b>Capital</b>	1,000	300	1,000	1,000	1,200			
<b>Total Input</b>	3,400	3,500	4,400	4,400	5,000			

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} 0.058 \cdot k_1^{0.35} \cdot k_2^{0.01} \cdot k_3^{0.10} \cdot k_4^{0.07} \cdot k_5^{0.08} \cdot l_1^{0.20} \cdot l_2^{0.01} \cdot l_3^{0.05} \cdot l_4^{0.04} \cdot l_5^{0.08} \cdot p_1^{1.2} \cdot p_2^{0.1} \cdot p_3^{0.4} \cdot p_4^{0.3} \cdot p_5^{0.3} \\ 0.012 \cdot k_1^{0.1} \cdot k_2^{0.12} \cdot k_3^{0.14} \cdot k_4^{0.12} \cdot k_5^{0.13} \cdot l_1^{0.05} \cdot l_2^{0.08} \cdot l_3^{0.07} \cdot l_4^{0.07} \cdot l_5^{0.13} \cdot p_1^{0.3} \cdot p_2^{1.3} \cdot p_3^{0.6} \cdot p_4^{0.5} \cdot p_5^{0.5} \\ 0.034 \cdot k_1^{0.05} \cdot k_2^{0.03} \cdot k_3^{0.32} \cdot k_4^{0.11} \cdot k_5^{0.11} \cdot l_1^{0.03} \cdot l_2^{0.02} \cdot l_3^{0.16} \cdot l_4^{0.06} \cdot l_5^{0.11} \cdot p_1^{0.2} \cdot p_2^{0.3} \cdot p_3^{1.4} \cdot p_4^{0.5} \cdot p_5^{0.4} \\ 0.046 \cdot k_1^{0.10} \cdot k_2^{0.02} \cdot k_3^{0.06} \cdot k_4^{0.32} \cdot k_5^{0.10} \cdot l_1^{0.06} \cdot l_2^{0.01} \cdot l_3^{0.03} \cdot l_4^{0.19} \cdot l_5^{0.10} \cdot p_1^{0.4} \cdot p_2^{0.2} \cdot p_3^{0.3} \cdot p_4^{1.4} \cdot p_5^{0.4} \\ 0.064 \cdot k_1^{0.07} \cdot k_2^{0.02} \cdot k_3^{0.09} \cdot k_4^{0.09} \cdot k_5^{0.32} \cdot l_1^{0.04} \cdot l_2^{0.02} \cdot l_3^{0.05} \cdot l_4^{0.03} \cdot l_5^{0.32} \cdot p_1^{0.2} \cdot p_2^{0.3} \cdot p_3^{0.4} \cdot p_4^{0.2} \cdot p_5^{1.3} \end{bmatrix} \cdot (15)$$

Equation 15 displays all primary factors and TFP elasticity of total products of sectors. As it is shown, due to relative substitutability among all primary inputs, the effect of a one *per cent* change in primary factors of a sector affects the products of all sectors. In addition, the effect of change in the TFP of a sector can be traced on total outputs of all sectors. For instance, one *per cent* increment in the level of TFP of Sector 1 leads to 0.2 *per cent* increment in total output of Sector 3.

**Table 2 | The MTRS of  $k_3$  for Different Levels of  $l_3$**

$l_3$	800	600	400	200	0
<b>Proposed model</b>	0.20	1.00	1.91	5.80	$\infty$
<b>Ghosh model</b>	1	1	1	1	1

Source: own calculation

Using Equation 12, Table 2 displays the *MTRS* of  $k_3$  for different levels of  $l_3$  in Sector 3. It measures the size of  $k_3$  which will be released for a unit increment in the size of  $l_3$  to protect the level of total outputs of Sector 3. As it is shown, the *MTRS* of  $k_3$  for  $l_3$  in the proposed model is dependent on the size of  $l_3$ , whereas using the Ghosh model, it is equal to 1 for all levels of  $l_3$ . In addition, in contrast to the Ghosh model, where it is possible to replace a primary factor instead of the other one perfectly, both primary factors are required in the production process of the proposed model.

**Table 3 | The Changes in the Levels of Products of Sectors Due to One Unit Change in the Size of Aggregated Primary Factors in Sector 1 Using the Ghosh and Proposed Models**

		$q_1^G$	$q_2^G$	$q_3^G$	$q_4^G$	$q_5^G$	$q_1^P$	$q_2^P$	$q_3^P$	$q_4^P$	$q_5^P$
$v_1$	1,560	3,400	3,500	4,400	4,400	5,000	3,400	3,500	4,400	4,400	5,000
	1,561	3,401.19	3,500.34	4,400.23	4,400.45	5,000.33	3,401.19	3,500.34	4,400.23	4,400.45	5,000.33
$\Delta q_j$		1.19	0.34	0.23	0.45	0.33	1.19	0.34	0.23	0.45	0.33
$v_1$	2,560	4,594.1	3,838.3	4,633.6	4,854.3	5,325.5	4,460.02	3,771.38	4,584.29	4,765.42	5,257.91
	2,561	4,595.3	3,838.6	4,633.9	4,854.8	5,325.8	4,460.98	3,771.60	4,584.44	4,765.72	5,258.12
$\Delta q_j$		1.19	0.34	0.23	0.45	0.33	0.95	0.22	0.15	0.30	0.21
$v_1$	3,560	5,788.21	4,176.52	4,867.29	5,308.59	5,650.90	5,343.18	3,963.62	4,711.25	5,025.37	5,436.94
	3,561	5,789.41	4,176.86	4,867.53	5,309.05	5,651.23	5,344.00	3,963.79	4,711.35	5,025.59	5,437.10
$\Delta q_j$		1.19	0.34	0.23	0.45	0.33	0.82	0.17	0.11	0.23	0.16

\*  $q_1^G$  to  $q_5^G$  refer to the level of products of Sectors 1 to 5 using the Ghosh model,  $q_1^P$  to  $q_5^P$  refer to the level of products of Sectors 1 to 5 using the proposed model, respectively.

Source: own calculations

The changes in the levels of aggregate primary factors of a sector on total products of all sectors are investigated through Equation. 14. Table 3 displays the changes in the level of products of sectors as a result of a unit of increment in the size of  $v_1$  in different levels of this factor. As it is shown, using Equation 2, increment in the level of  $v_1$ , has a constant effect on the level of products of sectors irrespective of the value of  $v_1$  in the Ghosh model, whereas it has decreasing effects in the proposed model. For instance, through the Ghosh model, one unit increment in the size of  $v_1$  leads to a 1.19 unit increment in the level of the products of Sector 1, irrespective of the value of  $v_1$ . Whereas, using the proposed model, the effects of one unit increment in the size of  $v_1$  on the level of products of Sector 1, vary with respect to the value of  $v_1$  from 1.19 to 0.82 units.

## 5. Conclusion

In this study, in order to improve the capability of the I-O model, a CD nonlinear supply-driven I-O model has been proposed. It has been demonstrated that it is possible to employ an I-O table for a nonlinear model. The model has a number of characteristics, which are generally more appropriate for production functions.

The proposed model replaces the less realistic characteristics of the Ghosh model with more appropriate ones. For instance, the perfect complementary characteristics between total intermediate inputs with total primary inputs, are replaced by relative substitutability of all production factor characteristics. In addition, the perfect substitutability characteristics of primary factors has changed to a relatively substitutable characteristics of these factors in the proposed model. These characteristics allow the proposed model to overcome the implausible problem of the Ghosh supply-driven I-O model.



The parameters of the proposed model can be employed to specify the conditions of the economy. Using the Solow residual of equations, it is possible to specify the effect of change in the TFP of a sector on total outputs of all sectors. It is also possible to study the effect of change in primary inputs on total products of sectors.

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