MAINTENANCE COMMITMENTS FOR MONOPOLIZED GOODS

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Abstract:

This paper highlights the monopoly firms' commitments for goods requiring high maintenance expenditure, such as elevators, televisions and computers. A guarantee time limit model to maintain these special goods is presented in this paper. Based on this model, several types of commitments with different guarantee time limits are compared under monopoly conditions. This paper finds that the guarantee pattern has no effect on the monopoly firm’s profits if all information is known to both the consumer and the monopolist. It is also shown that if a monopoly firm exaggerates its product quality claims in its advertisements, then it cannot meet its warranty guarantees. Industrial organizational theory is employed to analyze maintenance guarantees in this work.

Keywords: market structure, industrial organization, maintain, commitment, guarantee, price, game theory

JEL Classification: C61, C72, D4, L1

1. Introduction

For industrial organizational theory, market structure plays an extremely important role in determining price (Tirole, 1988; Akkoyunlu-Wigley & Mihi, 2006; Gaffard & Quere, 2006; Nie, 2010). Product properties also play a crucial role in determining firm behaviour and consumer behaviour (Church & Ware, 2000; Tesfatsion, 2001; Caves, 2007; Lambson & Phillips, 2007). This is particularly evident for durable and storable goods, with extensive literature available about storable goods under monopoly conditions (Dudine, Hendel & Lizzeri, 2006; Coase, 1972; Caves, 2007 Anton and Das Varma, 2005). All of these motivate us to further consider market behaviour for goods with special properties. In this paper, we consider goods with high expenditure to maintain under guarantee commitments.

Commitment of firms is exceedingly important, and has crucial effects on price. Based on data from the US, house consumption with commitment, for example, caused low

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prices (Krueger et al., 2008). Commitment was compared with non-commitment under monopolies for storable goods (see the interesting paper Dudine, Hendel & Lizzeri, 2006). For storable goods, price with commitment is lower than that without commitment under monopolies. The result was extended to other market structures (Nie, 2009). Commitment theory was further developed recently (Krueger and Uhlig, 2006). Price and quality relationships were also examined in online markets (Jin and Kato, 2006).

Goods requiring high expenditure to maintain, such as elevators, televisions, computers, and home appliances are exceedingly important in society, making it necessary to launch extensive research into these goods. The behaviour of firms and market prices may vary greatly with different patterns of guarantee time limits. This motivates us to consider guarantees for goods requiring high expenditure for maintenance.

Some research exists on maintenance guarantees. While most research supports a positive relationship between service guarantee and quality, some research questions this relationship (Heys and Hill; 2006) and references therein. In a recent paper, price-matching guarantees (PMGs) of retailers were considered, and the potential negative effects on consumer perceptions were investigated (Estelami, Grewal and Roggeveen, 2007). Economic research suggests that PMGs can support a mechanism of collusion among the retailers (Corts, 1997; Chen, 1995). The presence of PMGs by one retailer provides a disincentive to other retailers to lower their prices, since their price will be matched by the PMG-offering retailer.

Interactions between pricing and ethics were recently considered for two service industries (Indounas, 2008). In this paper, we aim to consider companies providing maintenance and repair services. Almost all maintenance guarantee research focuses on the business field and we hope to do further research on guarantees in industrial organization theory. We aim to consider market behaviour under a monopoly situation for goods requiring high expenditure for maintenance. We model the above goods and then analyze some theoretic properties. We aim to illustrate the profits of the monopoly firm and consumer utility under the guarantee time limit $T$ to maintain these goods.

There existed paper in the warranties about durable goods (Utaka, 2006). This paper puts emphasis on the guarantee time while Utaka’s work stressed whether to offer repair services.

This paper is organized as follows: The model is outlined and discussed in Section 2. Some analysis and main results are presented in Section 3, with a linear example given to illustrate the theoretic results. A situation where the quality of a good is exaggerated by the monopoly firm is considered in Section 4. Some remarks are provided in the final section.

2. The Model

We assume that there is a unique producer in some industry and the monopoly firm faces demand for a corresponding good that requires high expenditure to maintain. We further assume that the guarantee time period is exactly $T$. Namely, it is free to...
maintain this product in time $T$ after the consumer buys it. If the guarantee time limit expires, the consumer has to pay maintenance costs. As an extreme case, $T = 0$ is the case without a guarantee commitment.

The following notations are utilized in this paper:

$p$ and $q$ denote the price and quantity, respectively, of the good requiring high expenditure to maintain for the monopoly firm.

Let the cost for each product be $c_0$, where $c$ denotes the cost to repair each time per unit of the good. We assume that the probability to be maintained for each good observes an exponential distribution. The probability to repair at $t$ is $\varphi(t, \lambda) = \lambda e^{-\lambda t}$ for $t \geq 0$, where $\lambda$ is a constant dependent on the quality of the good and $\lambda^{-1}$ is the average life expectancy of the corresponding good. The parameter $\lambda$ depends on the technique of the monopoly firm. We further assume that $0 \leq T \leq \lambda^{-1}$ such that the guarantee time is shorter than the average life expectancy. Therefore, the probability to repair this good from $t = 0$ to $t = t_0$ is $\int_0^{t_0} \varphi(t, \lambda)dt = 1 - e^{-\lambda t_0}$. Actually, the life cycle of many types of electronic products and other durable goods observes this kind of distribution.

We assume that the cost to repair the corresponding good with quantity $s$ each time is $c(s) = cs$. We also assume that repairing is much cheaper than buying a new one. Consumer utility is quasi-linear in the consumption of good $q$ and money $m$. Namely, $U(q, m) = u(q) + m$. We further assume that $u$ is continuously differentiable. The consumer model is given as follows: Given any price $p$, the consumer chooses $q$ to maximize utility. The following utility maximization problem (UMP) is given:

$$\max_q u(q) - qp - c\int^\lambda_0 \varphi(t, \lambda)dt$$

Let $D(p)$ be the static demand function associated with $U$. Market clearing condition is always met or $q = D(p)$. Given the price, the monopolist aims to maximize its objective function or corresponding profits:

$$\max_p \pi(p, q) = pq - c\int^\lambda_T \varphi(t, \lambda)dt - c_0q.$$ 

In the above model, we further assume that both the monopoly firm and the consumer are rational. When they make decisions, the monopoly firm aims to maximize profits, and the consumer aims to maximize utility.

In addition, for the purpose of tractability, the linear expenditure to maintain and the linear cost function are employed in the above model. The results of this paper can be extended to the situation with restrictions of linear functions. The following assumptions are presented:

**Assumption 1** $u(q)$ is concave and twice differentiable in $q$, which guarantees the existence of the unique solution for the consumer. Furthermore, $u'(q) > 0$ for all $q$.

**Assumption 2** $D(p) > 0$.

**Assumption 3** $R(p) = \pi(p, D(p))$ is concave and twice differentiable in $p$, which guarantees the existence of a unique solution for the above model.
Assumption 1 and Assumption 3 manifest the existence of the unique solution to the above problem. Assumption 2 guarantees that the consumer consumes a positive quantity in the equilibrium state.

3. Main Results

In this section, we aim to consider the roles of the guarantee commitment. For convenience, we consider the situation in which the monopoly firm offers different types of guarantee commitment time limits. To simplify, we assume that there always exists an equilibrium solution to the above problems.

3.1 The Equilibrium

We first consider the equilibrium for the guarantee commitment $T (0 \leq T \leq \lambda^{-1})$ with high expenditures to maintain under a monopoly structure. When the monopoly producer provides a maintenance commitment with $T$ periods, the equilibrium prices are $p(T)$, along with $q = D(p,T)$. Under the commitment to maintain with time $T$, the following result therefore holds:

**Proposition 1** For the equilibrium demand $D(p,T)$ with the monopoly structure, we have the following results:

$$\frac{\partial D(p,T)}{\partial p} < 0, \frac{\partial D(p,T)}{\partial T} > 0.$$ \hspace{1cm} (3)

**Proof:** See the proof in Appendix.

The above conclusion manifests that consumer demand decreases when the price increases, which is a classic microeconomics result. Further, for a given price, if the guarantee time is prolonged, demand increases. On the contrary, the demand decreases.

We now consider the equilibrium price of the monopoly firm, with the equilibrium price denoted as $p(T)$. According to the corresponding consumer equilibrium $D(p,T)$, considering the first order optimal conditions of the monopolization problem (2), we have the following conclusion:

$$\frac{\partial D(p,T)}{\partial p} [p - c_0 - c \int_0^T \varphi(t,\lambda)dt] + D(p,T) = 0.$$ \hspace{1cm} (4)

We further denote

$$V(T) = \max_p pD(p,T) - cD(p,T)\int_0^T \varphi(t,\lambda)dt - c_0 D(p,T).$$ \hspace{1cm} (5)

Considering the properties of the equilibrium price $p(T)$, we therefore have the following result for the monopoly firm:

**Proposition 2** For the equilibrium price based on equilibrium demand $D(p,T)$, we have the following results:

$$\frac{dp(T)}{dT} > 0.$$ \hspace{1cm} (6)

**Proof:** See the proof in Appendix.
The above result illustrates that the guarantee time commitment has direct effects on the price of the corresponding good under a monopoly structure. Namely, longer guarantee times result in higher prices.

According to the results in Proposition 1 and 2, longer guarantee times have two effects. Firstly, it causes demand to increase and, secondly, longer guarantee times result in higher prices. Guarantee times therefore present a dilemma, making it important to determine the optimal guarantee time.

### 3.2 Monopoly Profits and Social Welfare

We now consider monopolization profits under an equilibrium state. According to (4), the following result holds:

**Proposition 3** For the monopoly firm, the guarantee time satisfies the following formulation:

\[ \frac{dV}{dT} = -\left( \frac{\partial D}{\partial T} \frac{\partial T}{\partial p} + \lambda e^{-\lambda T} \right)D(p, T) = 0. \]  

Namely, the guarantee time has no effect on the monopoly firm’s profits.

**Proof:** By the envelop theorem to (5), we then have the following conclusion:

\[ \frac{dV}{dT} = \frac{\partial D}{\partial T} \left[ p - c - c \int_{0}^{T} \varphi(t, \lambda)dt \right] - \lambda c D(p, T)e^{-\lambda T}. \]  

(8)

Combined (8) and (4), the following equation thus holds:

\[ \frac{dV}{dT} = -\frac{\partial D}{\partial T} \frac{\partial T}{\partial p} D(p, T) - \lambda c D(p, T)e^{-\lambda T}. \]  

(9)

Furthermore, according to the proof of Proposition 1 in the Appendix, we have

\[ \frac{\partial D(p, T)}{\partial p} = \frac{1}{\partial f/\partial q} = \frac{1}{\partial^2 u/\partial q^2}. \]

\[ \frac{\partial D(p, T)}{\partial T} = -\lambda ce^{-\lambda T}. \]

We therefore have

\[ \frac{\partial D}{\partial T} \frac{\partial T}{\partial p} = -\lambda ce^{-\lambda T}. \]  

(10)

(7) therefore holds by virtue of (9) and (10). The result is immediately obtained and the proof is therefore complete.

**Remarks:** According to (7), the guarantee time for the monopoly firm has no effect on profits if information on product quantity, cost and other factors are known both to the...
monopoly firm and to the consumer. Namely, when the parameters \( \lambda, c, c_0 \), the profit function and the utility function are all known not only to the monopoly firm but also to the consumer, the guarantee time has no effect on the monopoly firm’s profits.

We now consider the demand function \( D(p(t), T) \), with the following result holding:

**Proposition 4** If \( u(q) \) is a quadratic function, we then have the following result

\[
\frac{dD(p(T), T)}{dT} = 0. \tag{11}
\]

**Proof:** See the proof in Appendix.

Under the hypothesis of the concave quadratic, we find that demand has no relation to the guarantee time. In the general situation, this result may be violated.

We here consider social welfare and the following social welfare function is given:

\[
W(T) = \pi(p(T), T) + D(p(T), T) \tag{12}
\]

Under the equilibrium price, we consider the effects of the guarantee time. Combined with the conclusions in Proposition 3 and 4, we immediately have the following conclusion:

**Proposition 5** For the above social welfare function, if \( u(q) \) is quadratic, the guarantee time has no effect on social welfare.

We now provide a linear example to illustrate the above theory:

**Example 1** The following linear example is presented to illustrate the above theory: Let \( c = 10, \ c_0 = 20, \ \lambda = 0.1 \) and \( 0 \leq T \leq 10 \).

We assume that the demand function is given by the following formulation:

\[
q = 100 - p - 10e^{-0.1T}
\]

\[
\pi(p, T) = q[p - 20 - 10(1 - e^{-0.1T})] = q(p - 30 + 10e^{-0.1T}).
\]

Considering the profit maximizing problem of the monopoly firm, \( \max_p \pi(p, T) \), by virtue of the first order optimal conditions we therefore have \( p(T) = 65 - 10e^{-0.1T} \) and the corresponding profits of the monopoly firm are \( V(T) = 35^2 \). Furthermore, \( D(p(T), T) = 35 \). Thus, the monopoly firm’s profits and demand have no relationship with the guarantee commitment. We here consider the corresponding social welfare function \( W(T) = \pi(T) + q(T) = 35^2 + 35 \). The guarantee commitment therefore has no effect on social welfare in this linear example.

All these results are highly consistent with the above theory, from Proposition 1 to Proposition 5, in this section.
4. Situations with Exaggerated Quality of Goods

In the above theory, product quality is known both to the consumer and to the monopoly firm. However, there actually exists private information of the monopoly firm. Here we assume that the quality is exaggerated by the monopoly firm. The consumer think that the \( \bar{\lambda}^{-1} \) is the average life expectancy of the corresponding good, where \( \bar{\lambda}^{-1} > \lambda^{-1} \) or \( \bar{\lambda} < \lambda \). Given any price \( p \), consumers choose \( q \) to maximize their utility. Namely,

\[
\max_q U(q, m) - pq - cq \int_{T}^{\bar{\lambda}^{-1}} \varphi(t, \lambda)dt
\]  

(13)

Consider the model (13) and (2), the results similar to those Proposition 1 and Proposition 2 are also obtained and the proof is similar to those in Proposition 1 to Proposition 2.

\[
\frac{\partial D(p, T)}{\partial p} = \frac{1}{u} < 0, \tag{14}
\]

\[
\frac{\partial D(p, T)}{\partial T} = -\frac{\bar{\lambda}ce^{-\bar{\lambda}T}}{u} > 0, \tag{15}
\]

It is obvious that (6) should hold under the situation in this section. We here consider the profits of the monopolization with the guarantee time limit \( T \).

**Proposition 6** For the monopoly firm, the relationship between guarantee time and profits satisfies the following formulation:

\[
\frac{dV}{dT} = -(\frac{\partial D}{\partial T} \frac{\partial T}{\partial p} + \lambda ce^{-\lambda T})D(p, T) < 0. \tag{16}
\]

Namely, \( T = 0 \) is the optimal strategy for the monopoly firm.

**Proof:** On one hand, according to the properties of the function, consider the function \( f(x) = xe^{-Tx} \). \( f'(x) = e^{-Tx} - Txe^{-Tx} = (1 - Tx)e^{-Tx} > 0 \) if \( 1 - Tx > 0 \). We therefore have that \( f(x) = xe^{-Tx} \) is increasing if \( x < T^{-1} \). Namely, \( \bar{\lambda}ce^{-\bar{\lambda}T} - \lambda ce^{-\lambda T} < 0 \) because \( \lambda > \bar{\lambda} \) and \( \lambda T < 1 \).

On the other hand, similar to the methods in Proposition 3, we have the following result different from (7).

\[
\frac{dV}{dT} = -(\frac{\partial D}{\partial T} \frac{\partial T}{\partial p} + \lambda ce^{-\lambda T})D(p, T) = (\bar{\lambda}ce^{-\bar{\lambda}T} - \lambda ce^{-\lambda T})D(p, T) < 0. \tag{17}
\]

Namely, \( T = 0 \) is the optimal strategy for the monopoly firm that provides exaggerated information regarding its goods. The result is immediately obtained and the proof is therefore complete.
(16) illustrates that the monopoly firm is not willing to provide a maintenance guarantee if the quality is exaggerated by the monopoly firm. Similar to the above result in Proposition 6, we have the following result:

**Proposition 7** If \( u(q) \) is a quadratic function, we then have the following result

\[
\frac{dD(p(T), T)}{dT} < 0 .
\]

The proof is similar to that in Proposition 4 and the proof is omitted. The above result manifests that prolonging guarantee times results in reduced demand if the quality is exaggerated.

Under equilibrium pricing, we consider the effects of the guarantee time with exaggerated quality. Combined with the conclusions in Proposition 6 and 7, we then have the following conclusion:

**Proposition 8** If \( u(q) \) is quadratic, a longer guarantee time results in a social welfare decrease.

We here give a linear example to illustrate the above theory, in which the monopoly firm exaggerates the quality:

**Example 2** The following linear example is given to illustrate the above theory. Let \( c_0 = 10, \quad c = 20, \quad \lambda = 0.1, \quad \lambda = 0.05 \) and \( 0 \leq T \leq 10 \). We assume that the demand function is given by the following formulation.

\[
q = 100 - p - 10e^{-0.05T}
\]

\[
\pi(p, T) = q( p - 20 - 10(1 - e^{-0.17})) = q(p - 30 + 10e^{-0.17}).
\]

Considering the profit maximizing problem of the monopoly firm, \( \max_p \pi(p, T) \), by virtue of the first order optimal conditions, we therefore have \( p(T) = 65 - 5e^{-0.017T} - 5e^{-0.057T} \) and the corresponding profits of the monopoly firm are \( V(T) = (35 + 5e^{-0.017T} - 5e^{-0.057T})^2 \).

Thus, the optimal guarantee time is \( T = 0 \). The demand function is \( D(p(T), T) = 35 + 5e^{-0.017T} - 5e^{-0.057T} \) and demand decreases with longer guarantee times.

All of these results are consistent with the above theory in this section. According to the above results and the corresponding example, if the monopoly firm exaggerates product quality via advertisements, it will not need to provide a guarantee to maximize its profits.
5. Concluding Remarks

In this work, the theory of goods requiring high maintenance expenditure under monopoly conditions is considered, and the corresponding results are obtained. Several types of guarantee maintenance commitments are analyzed and compared.

For this paper’s model, some assumptions seem to be ideal. These assumptions can be immediately extended to general situations. However, problems may be more complicated under general situations.

We find that if a monopoly firm exaggerates the quality claims of its product in its advertising, then it cannot meet its warranty commitments. It is therefore important to provide as much product information as possible in evaluating economic problems. In reality, all consumers expect longer guarantee maintenance times because private information of the firm may exist. Alternatively, a longer maintenance guarantee time can improve consumer confidence. All of these are successfully explained in this paper under the monopoly situation. Furthermore, this also supports a rational explanation of the positive relationship between service guarantee and quality.
Appendix

Proof of Proposition 1

For the general situation, it is very difficult to employ comparative static analysis methods. The implicit function theorem is thus employed. We first consider the first order optimal conditions and then analyze the first order optimal conditions by implicit function techniques. Consider the first order optimal conditions to (1), we have

\[ f = \frac{du}{dq} - p - c \int_{\tau}^{\lambda} \varphi(t, \lambda) dt = \frac{du}{dq} - p + c(e^{-1} - e^{-\lambda T}) = 0. \]

According to the concave of \( u(q) \), we have that \( \frac{\partial f}{\partial q} = \frac{d^2 u}{dq^2} < 0 \) and, furthermore, \( \frac{\partial f}{\partial q} \neq 0 \) holds. By virtue of the implicit function theorem in mathematic analysis, there exists a unique solution \( q(p, T) \) which is continuous and, furthermore, \( q(p, T) \) is continuously differential. The following conditions are met

\[ \frac{\partial D(p, T)}{\partial p} = -\frac{\partial f / \partial p}{\partial f / \partial q} = \frac{1}{\partial f / \partial q} < 0, \]

\[ \frac{\partial D(p, T)}{\partial T} = -\frac{\partial f / \partial T}{\partial f / \partial q} = -\lambda ce^{-\lambda T} > 0. \]

Therefore, (3) holds. The result therefore holds and the proof is complete.

Proof of Proposition 2

We also consider the first order optimal conditions and then analyze the first order optimal conditions by implicit function techniques. Consider the first optimal conditions to (2), the following formulation holds.

\[ g = \frac{\partial D(p, T)}{\partial p} \left[ p - c_0 - c \int_{0}^{T} \varphi(t, \lambda) dt \right] + D(p, T) = 0. \]

According to the concavity of \( R(p) \), we have that \( \frac{\partial g}{\partial p} < 0 \) and, furthermore, \( \frac{\partial g}{\partial p} \neq 0 \) holds. By virtue of the implicit function theorem in mathematic analysis, there also exists a unique solution \( p(T) \) which is continuous. Furthermore, \( p(T) \) is continuously differential.

According to the formulation of \( \frac{\partial D(p, T)}{\partial p} = \frac{1}{\partial f / \partial q} \) (see the proof in Proposition 1), we have \( \frac{\partial^2 D}{\partial p \partial T} = 0 \) and the following equation therefore holds.
Furthermore, according to the explicit function theorem, the following conditions are met

\[
\frac{dp(T)}{dT} = -\frac{\partial g}{\partial T} > 0.
\]

Therefore, (3) holds. The result therefore holds and the proof is complete.

**Proof of Proposition 4**

On one hand, if \( u(q) \), is both concave and quadratic, \( D(p,T) \) is linear in \( p \) and the following equation holds

\[
\partial \left[ \frac{\partial D(p,T)}{\partial p} \right] = 0.
\]

We hence obtain

\[
\pi = \frac{\partial \left[ \frac{\partial D(p,T)}{\partial p} \right] [p - c_0 - c^T \phi(t,\lambda)dt] + D(p,T)}{\partial p} = \frac{\partial D(p,T)}{\partial p} + \frac{\partial D(p,T)}{\partial p} = 2 \frac{\partial D(p,T)}{\partial p} = 2 \frac{\omega}{u}.
\]

On the other hand, according to the derivatives of the function \( D(p,T, T) \), we have

\[
\frac{dD(p(T),T)}{dT} = \frac{\partial D}{\partial p} \frac{dp}{dT} + \frac{\partial D}{\partial T} = \frac{1}{u} \left( \frac{dp}{dT} - \lambda ce^{-\lambda T} \right) = \frac{\lambda ce^{-\lambda T}}{u} \left( \frac{2}{u \pi} - 1 \right) = 0.
\]

The result in this proposition is therefore obtained and the proof is complete.
References


