
Pavel Potužák*

Abstract

This article demonstrates that the pure time-preference theory of Ludwig von Mises is inconsistent. A productivity element is studied in the Fisher model, and it is shown that time preference is neither a necessary nor a sufficient condition for the existence of interest. An attempt is also made to reconcile the Austrian theory with the neoclassical theory of interest. It is suggested that the key difference lies in the definition of interest as such, and it is concluded that the Austrian theory is only a special case of a more general neoclassical framework.

Keywords: time-preference theory, productivity of capital, theory of interest, zero lower bound, Fisher, Mises

JEL Classification: B25, E43, B53

1. Introduction

Monetary policy of zero nominal interest rate in the majority of the most developed countries accompanied by a low but positive inflation rate depressed the real interest rate to a negative region. Since there is no signal that this policy will be terminated in the foreseeable future, the recent period may become known as a decade in which future goods were exchanged with premium for present goods. In real terms, savers are today prepared to voluntarily exchange 100 units of present goods for 99 units delivered in the future.

Ludwig von Mises ([1949], 1996) and his followers in the pure time-preference school claimed that the interest rate in the economy must be always positive due to the *a priori* existence of positive time preference. A zero or negative interest rate is a signal of an artificial monetary manipulation of the central bank or of a government coercive action. The former should end up in a galloping inflation due to the gap between the actual and the natural rate of interest. The latter may provoke consumption of capital in the economy. However, the development in recent years does not seem to confirm Misesian predictions. One possible explanation might be that nowadays the natural rate of interest is negative, which is in direct contradiction with the majority of writings in the Austrian tradition.

Although Irving Fisher was considered by many economists a pure time-preference theorist (Seager, 1912; Hayek, 1941), he introduced examples in which the natural rate of interest is zero or even negative (Fisher, 1930). This paper will extend the original Fisherian approach, and it will demonstrate that the Mises theory is only a special case of a more general neoclassical framework.

* Pavel Potužák Department of Economics, University of Economics Prague, Czech Republic (pavel.potuzak@vse.cz).

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The paper proceeds as follows. In Section 2, an objective element is introduced into the theory of interest. Three famous Fisherian examples clarify that the productivity of capital may be the crucial determinant of the interest rate in the economy, regardless of the size of time preference. A model of constant marginal productivity of capital is presented in which the time preference (in the sense of the marginal rate of substitution between present goods and future goods) is adjusted to the ongoing rate of interest that may, however, reach any value – positive, zero, or negative. This section demonstrates that a negative interest rate might exist simultaneously with a positive time preference (in the sense of the subjective discount rate).

In the next part, the response and defence of modern protagonists of the pure time-preference theory (PTPT) are studied. It is shown that the critical controversy between the Austrian and the neoclassical approach arises in the definition of interest as such. Concepts of nominal interest and real interest are introduced, which are mainly designed to pin down crucial differences between these two schools of economic thinking. An attempt is made to reconcile the two approaches. However, it is suggested that the PTPT is just a special theory within a more general neoclassical framework as the real approach, associated with the latter school, seems to be the superior one.

Section 3 also uncovers that the examination of the nominal and real approach has strong interconnections to the dynamic efficiency discussion. It is shown that a positive value difference between output and expended inputs that the authors of the PTPT consider to be at the centre of the interest theory and that can be, according to them, explained only by an *a priori* existence of positive time preference, may emerge in the economy with constant money supply and constant marginal productivity of capital only if such an economy is dynamically efficient. The last part concludes.

2. Productivity Element and the Irrelevance of Time Preference

The pure time-preference theorists deny that the productivity of capital should play any role in the explanation of the interest phenomenon. Mises ([1949], 1996) frequently claimed that only time preference determines the originary (natural) interest. Since present satisfaction of a want (*e.g.* hunger) is always preferred to later satisfaction of the same want (*ibid.*, p. 483), the time preference is a ‘categorical requisite of human action’ (*ibid.*, p. 484). This results in the fact that the natural rate of interest ‘cannot disappear as long as there is scarcity and therefore action’ (*ibid.*, p. 528). According to Mises, in the world without time preference, man ‘would always accumulate, he would never consume and enjoy’ (*ibid.*, p. 484).

Mises’s successor Murray Rothbard ([1962], 2004, p. 424) claimed that any increase in the productivity of capital will result only in temporary profits. If the physical productivity of capital goods increases, their market value will eventually rise, but the value difference between capital goods and the final output of consumption goods will return to the previous level that is dictated solely by the time preference.

This idea can be traced back to the classic work of Irving Fisher (1930). Moreover, Fisher (1930) presented an example in which the interest rate must be necessarily zero. His reasoning is so tempting on the one hand, and so explicitly at variance with the Mises theory, on the other hand, that it will be the central issue of the following analysis.

Fisher (1930, 186 ff.) introduced a story of shipwrecked sailors left with a given stock of non-perishable hard-tacks. This stock of consumption goods cannot be increased,

so the only problem the sailors face is as follows: what is the optimal allocation of hard-tacks over their lives?

Sailors may consume in the future only if some amount of hard-tacks is saved in the present. Saving cannot be used for productive investment purposes, hence one present non-consumed hard-tack can “produce” one hard-tack ready for consumption in the future. Fisher claimed that the only equilibrium exchange ratio between present hard-tacks and future hard-tacks is one. In other words, in this economy no interest on hard-tacks will emerge, which is in direct contradiction with the fundamental statement of the Mises theory.

According to Fisher, if the interest on hard-tacks was positive, no sailor would be willing to borrow. Nobody would accept borrowing 10 hard-tacks today and returning 11 hard-tacks in the future, because the same increase in present consumption could be achieved by a reallocation of the sailor’s own stock - simply 10 hardtacks would be consumed today instead of in the future. On the other hand, everybody would be willing to lend for the opposite reasons. This obvious imbalance should rapidly reduce the interest rate back to zero.

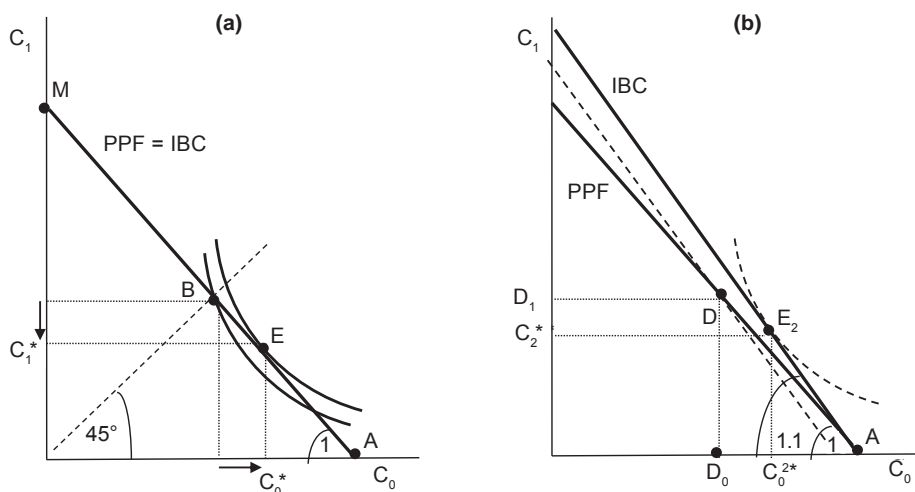
Fisher also concluded that the optimum marginal rate of substitution (MRS) of every sailor must be so adjusted as to make his subjective intertemporal valuation of hard-tacks consistent with the objective reality. The optimum MRS must be one - every sailor must shape his consumption stream in such a way that at the margin, the present hard-tack will be valued the same as the future hard-tack. Hence, there will be no discount on future goods.

At this point, we will demonstrate that Fisher’s reasoning is superior to the Mises theory. Consider a representative sailor who lives just in two periods. This sailor prefers the given satisfaction earlier rather than later, so if the first need on his value scale is to eat, the second one is to feed his dog (Böhm-Bawerk, [1888], 1891, p. 146), the third one to feed his cat, *etc.*, he will use the first hard-tack for eating now, the second hard-tack to feed his dog now, the third hard-tack to eat tomorrow, the fourth hard-tack to feed his cat today, the fifth hard-tack to feed his dog tomorrow, *etc.* This sailor perfectly meets the Misesian requirement of the superiority of present satisfaction.

Even though every sailor in this economy prefers present satisfaction to future satisfaction, the interest rate must be necessarily zero. According to Mises ([1949], 1996), the combination of positive time preference and zero interest rate is absolutely unthinkable, because on the unhampered market positive time preference must be always reflected in the positive rate of interest.

Figure 1 shows preferences of a representative sailor. The superiority of present gratification is reflected in the positive subjective discount rate, which in turn leads to the fact that the slope of the indifference curve at the diagonal line exceeds one (Ghez and Becker, 1975, p. 9). His stock of hard-tacks is depicted at point A. Because hard-tacks can be easily moved to the future, his resource constraint (or his production possibility frontier PPF, or a very degenerate investment opportunity line) is linear with the slope one (in absolute value). His resource constraint must perfectly coincide with his intertemporal budget constraint (IBC) because the market interest rate on this desert island (in terms of hard-tacks) is necessarily zero (see panel (a)).

Figure 1 | Optimum of a Shipwrecked Sailor on a Desert Island in the Hard-tack Economy



Source: Own construction

Let us discuss the reason for zero interest in more detail. If the interest rate in this economy was positive, for example 10%, no hard-tack from the entire fund A would be retained in the stock for the future. For a 10% interest rate and a linear PPF with the slope one (in absolute value), point A would generate the maximum present value of assets, as can be seen by comparing point A with point D_0 in panel (b) of Figure 1. If the number of $(A - D_0)$ hard-tacks were left in the stock, the present value of the sailor's assets (or the present value of the income stream D_0, D_1 represented by point D on the PPF in panel (b)) would be lower.

Thus, assuming a positive interest rate, no sailor from the population N would store a single hard-tack in his stock. Each (non-consumed) piece would be offered on the intertemporal market for a 10% interest rate. Only this decision would maximise the present value of his assets. However, in such a case, the total demand for present hard-tacks in the economy ($N \times C_0^*$) would fall short of the total supply of present hard-tacks ($N \times A$). Alternatively, it can be said that the net *per capita* supply of present hard-tacks ($A - C_0^{2*}$) could not find the corresponding net demand. As a result, the interest rate must decrease to zero to equalise the demand and supply. It must decline to equilibrate the intertemporal market.

As can be seen in panel (a), only a zero interest rate will eliminate the excess of present hard-tacks on the intertemporal market. The IBC will then perfectly coincide with the PPF. The demand for present hard-tacks will be equal to the supply of present hard-tacks even though they might not be traded on the intertemporal market. At the individual level, C_0^* hard-tacks will be consumed in the present, and $(A - C_0^*)$ will be retained in the stock for future consumption C_1^* .

Panel (a) in Figure 1 shows that the optimum of a sailor cannot be at point A, *i.e.* all hard-tacks are not consumed in the present even in the situation of zero interest and positive time preference (in the sense of the subjective discount rate ρ). The necessary break is performed by the tendency to equalise marginal utilities. A one-way shift of all goods to one particular period would radically reduce marginal utility in this period.

In other words, needs of very low intensity would be satisfied in this period at the expense of needs in other periods. And this cannot be optimal.

The optimum does not lie at point B either, where the consumption stream is perfectly smoothed. The reallocation of hard-tacks will continue until the MRS is equal to one (the same value as the marginal rate of transformation in this case). The optimum will be at point E at which the present hard-tack will be valued the same as the future hard-tack.

In this economy, each sailor has a positive subjective discount rate ($\rho > 0$) - the slope of the indifference curve at the 45° line is higher than one - which may be called the time preference in sense two (Murphy, 2003). On the other hand, the subjective exchange ratio between present goods and future goods, reflected in the MRS, might be called the time preference in sense one. At the optimum of a shipwrecked sailor in the hard-tack economy, this notion of time preference is zero ($MRS^* - 1 = 0$).

In Fisher's example, the market interest rate is not affected by sailors' subjective discount rates. Regardless of their impatience, the interest rate must be zero. The consumption path of each sailor will be so adjusted that his time preference in sense one ($MRS - 1$) will be depressed to zero as well. In other words, it will be shaped such that at the margin, the present hard-tack will be subjectively valued the same as the future hard-tack (in the sense $MRS^* = 1$, even though $\rho > 0$). The subjective discount rate ρ of each sailor (and the intertemporal elasticity of substitution in consumption that determines the curvature of the indifference curves) will only fix the optimum shape of the consumption stream over time, *i.e.* the specific position of point E on the budget line. However, it must be stressed that the budget line itself is determined by the objective phenomena - the initial stock of hard-tacks and their zero marginal productivity.

Fisher (1930) in his work offered another example of the irrelevance of time preference. In the second story, the saved goods exhibit positive rather than zero physical productivity. Fisher envisioned a herd of sheep that, if saved and properly invested, would increase future output of sheep by a constant per cent. According to Kirzner ([1993], 2011), Paul Samuelson speculated about a similar situation - each seed of rice, if not consumed, may (without any further costs) provide 1.1 seeds in the future.

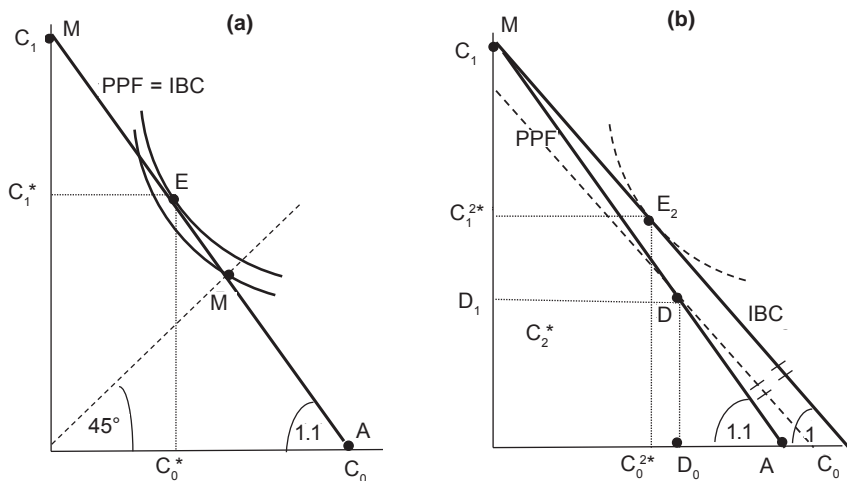
Compared with the hard-tack economy, the marginal productivity of capital in the 'rice economy' is positive. Capital has a form of the saved rice (or sheep), and it has a productive power to increase future output of consumption goods. Obviously, this example is elementary and highly stylized - both capital and consumption goods are represented by the same commodity. As such, it is a typical neoclassical one-good economy. However, even this example may provide us with important insights.

Another simplification in this example is that the marginal product of capital does not diminish. Every additional seed of rice invested provides a constant net return of 10%, regardless of the number of seeds invested before. The only equilibrium interest rate (in terms of rice) in this economy is 10%, as the following discussion demonstrates.

The resource constraint (PPF) of each sailor can be represented by a linear line with the slope of 1.1. Panel (b) in Figure 2 illustrates that if the market interest rate was lower than 10%, for example 0%, the intertemporal budget constraint (IBC) would be flatter than the PPF, and all present seeds of rice should be invested. By investing the entire stock, the producer would maximise the present value of his assets, as can be seen by comparing the present value of point M with point D, at which only $(A - D_0)$ seeds of rice would be planted in the present. Thus, all sailors (N) should invest their entire stock A. However,

in such a case, the demand for present goods $(C_0^{2*} + A) \times N$ would exceed the supply of present goods $A \times N$. Alternatively, it can be said that the aggregate investment $A \times N$ would fall short of aggregate saving $(A - C_0^{2*}) \times N$. At the individual level, the *per capita* net demand for present goods C_0^{2*} could not find the corresponding net supply. As a result, the interest rate r should rise to the point at which the shortage of present goods is completely eliminated.

Figure 2 | Optimum of a Shipwrecked Sailor in the “Rice” Economy



Source: Own construction

In the intertemporal equilibrium, the intertemporal budget constraint will coincide with the resource constraint, both having the slope of $(1 + r) = 1.1$. As can be seen in panel (a), the optimum MRS of every sailor must be 1.1 as well. Only at point E, the total demand for present goods due to consumption $N \times C_0^{2*}$ and investment $N \times (A - C_0^{2*})$ will be equal to the total supply of present goods $N \times A$. Total investment will be equal to total saving, and all sailors will also achieve maximum utility.

The optimum of each sailor can be found using the well-known condition that the interest rate r must be equal to the time preference (in sense one), which is defined as $\varepsilon \equiv \text{MRS} - 1$ (see Appendix 1). In the rice economy, the marginal rate of time preference (ε) in optimum is 10%. Furthermore, the optimum allocation of consumption of rice over time depends on the intertemporal preferences of each sailor. Figure 2 represents a sailor who is relatively patient - the slope of the indifference curve at the diagonal line is lower than the slope of the budget constraint. This sailor willingly consumes more rice in the future. As can be seen from equation (A1_17) in Appendix 1, if the interest rate r exceeds the subjective discount rate ρ , future consumption will exceed present consumption. The optimum path of consumption is increasing, or we may say that the optimum growth rate of consumption is positive.

There might be a sailor with the subjective discount rate of exactly 10%. This particular sailor will perfectly smooth his consumption stream because $r = \rho$. However, even if all sailors were of this type, it would not be correct to claim that the interest rate in this economy is determined by their time preference (in sense two, *i.e.* ρ). For any value

of ρ , the market interest rate in this economy is solely determined by the productivity of capital. It must be 10%, regardless of the impatience of sailors. Parameter ρ will only affect the optimum path of consumption over time.

Another important observation derived from this example is as follows. Suppose that the given satisfaction is not preferred as soon as possible, so the individual is indifferent about the moment of gratification of the given need (*i.e.* hunger). In mathematical terms, suppose that the subjective discount rate is zero. This assumption implies that the slope of the indifference curve at the diagonal line is one. However, a similar diagram, as is shown in Figure 2, may illustrate the optimum of this very patient sailor. Mises ([1949], 1996) claimed that the positive interest rate and zero time preference will lead the consumer to postpone every unit of consumption goods to the future. In other words, everything should be saved. As can be seen, this is not accurate, at least in the two-period model. The reason lies in the fact that the consumer's optimum is quite close to consumption smoothing because marginal utilities of consumption in each period should not be very far off from each other. In each period, wants of similar urgency are to be satisfied. Thus, the allocation of consumption goods over time will follow this requirement.¹

The positive interest rate is therefore perfectly consistent with zero time preference (in sense two). The example of rice economy also shows that there might exist time preference in sense one ($MRS > 1$) even if the time preference in sense two is absent ($\rho = 0$). Thus, people in this economy do not prefer the given want to be gratified as soon as possible, yet they do prefer a marginal present good over a marginal future good. Their time preference in the first sense is $\varepsilon = 10\%$. This endogenous meaning of time preference is perfectly adjusted to the ongoing rate of interest that is solely determined by the productivity of capital (Brown, 1913; Knight, 1941).

The previous two examples proved that the time preference in the Misesian sense (the superiority of present satisfaction) is neither a necessary nor a sufficient condition for the existence of interest. In the example with hard-tacks, we observed a zero market interest rate even when people preferred the given satisfaction to arrive as soon as possible (ρ was positive). In the example with constant and positive marginal productivity of capital (rice), the interest rate was positive even in the economy where people were indifferent about the time at which the given want would be satisfied (ρ was zero).

We will conclude this section with a theoretical possibility that is unthinkable in the Misesian system - a negative originary (or natural) interest. Fisher's third famous example is about shipwrecked sailors endowed with perishable figs (1930, p. 191). Suppose that the rate of their decay is 10% *per year*.

1 Surprisingly, this conclusion was made by young Hayek ([1927], 1984) in his very early paper. However, if the time horizon is extended to infinity, the Misesian argument must be reconsidered anew. The problem with the two-period model is that implicitly, there is an infinite time preference (in sense two, *i.e.* $\rho \rightarrow \infty$) in all periods following the death of the representative consumer (*i.e.* in our timing at $t = 2, t = 3, \dots$). Thus, we are inconsistent in saying that the consumer has no time preference in sense two (*i.e.* $\rho = 0$). This statement holds only as regards one future period. All the other periods are infinitely discounted. Hence, the attack against the Mises theory that claims that all consumption will be postponed to the future loses its power because there is an infinite, not zero, time preference in the other periods. However, it can be shown that even in the infinite horizon model, the simultaneous existence of positive interest rate and zero time preference (in sense two) will not result in a complete postponement of the act of consumption to the indefinite future if the elasticity of substitution is low enough (Olson and Bailey, 1981).

The solution of this problem is the same as with hard-tacks and rice. In a two-period model, even if no figs are consumed today, their stock will fall by 10% in one year. In this economy, the only equilibrium exchange rate between present figs and future figs is 0.9. In other words, the only equilibrium interest rate is -10% (*i.e.* minus 10%). If it was, for example, 0%, everybody would be prepared to offer present figs to obtain the same amount of figs in the future. On the other hand, no one (honest) would be willing to borrow ten present figs and deliver the same amount in the future. A mere reallocation of his own figs to present (*i.e.* eating additional ten figs today instead of in the future) would obviously represent a better option. Thus, the resulting surplus of the supply of present figs over their demand on this intertemporal market will necessarily reduce the interest rate to the equilibrium level of -10% . Hence, if only nine out of ten figs survive to the future, their subjective intertemporal valuation must follow this ratio. Figs will be allocated over time such that the optimum MRS of every sailor is 0.9, regardless of his subjective discount rate ρ .

We can conclude that in this particular example, a negative interest rate will emerge in the economy even when sailors exhibit positive time preference (in sense two). In their optimum, they have a negative time preference in sense one ($\text{MRS} - 1 = \varepsilon = -0.1$), whereas in sense two, it is positive ($\rho > 0$). In other words, sailors do prefer the given want to be satisfied as soon as possible, yet the subjective (and objective) exchange ratio between present goods and future goods is 0.9. Every man in this economy is willing to exchange one present fig for nine tenths of a future fig. Thus, present goods are valued less than future goods. Moreover, the interest rate is determined by an objective element - by the rate of deterioration of figs.

3. Physical Productivity versus Value Productivity, Real Interest versus Nominal Interest

Modern proponents of the pure time-preference theory seem to be aware of the Fisherian approach presented in the previous section.² Garrison (1979), for example, objected against the hard-tack economy that it is ridiculous to talk about the (zero) interest rate, intertemporal markets, intertemporal decisions, and even about human action as such in this highly stylised economy. After all, there is no intertemporal exchange, since no hard-tacks are traded among sailors. There are no borrowers and no lenders.

It should be admitted that there is no intertemporal market in the hard-tack economy. However, this does not mean that zero interest rate is not the market equilibrium. The intertemporal exchange is eliminated because zero market interest rate gives the same return as storing hard-tacks in the stock. Even a tiny deviation of the market interest rate from the zero equilibrium would provoke a creation of a true intertemporal market, but with a gigantic imbalance. If the interest rate was positive, the market would be flooded with present hard-tacks offered in exchange for a higher amount of future hard-tacks. A negative interest rate would lead to the opposite tendency. As a result, only zero interest rate guarantees equilibrium in the intertemporal market even though this market as such will not exist. Furthermore, it is inaccurate to claim that there is no human action. Sailors

2 See, for example, Walter Block (1978), Roger Garrison (1979), Israel Kirzner ([1993] 2011), Peter Lewin (1997), Jörg Guido Hülsmann (2002), Paul Cwik (2004), Jeffrey M. Herbener (2011; 2013), and Mihai Vladimir Topan and Cristian Păun (2013).

must decide upon the optimum allocation of hard-tacks over time, which would become even more obvious if the time horizon was extended to T periods.

Garrison ([1988], 2011, p. 170) also objected that in examples with positive and constant marginal productivity, the phenomenon of interest is confused with the growth rate of output. However, it can be easily shown that this remark is also inaccurate. Even though the net marginal product of capital in the rice economy is 10%, this does not imply that output of rice in the next period will be higher by the same percentage. As can be seen from equation (A1_17) in Appendix 1, the eventual increase in the (demanded) output depends on the subjective discount rate ρ and the intertemporal elasticity of substitution $1/\theta$. Since the MPK is constant and the investment opportunity line is linear, the cost curve in this economy is a horizontal line at the given relative price between future goods and present goods (*i.e.* $1/1.1$ in our case). Because the perfectly elastic supply curve can meet any demand for the given relative price, the intertemporal demand (given by the Euler equation A1_17) determines the optimal ratio between present output and future output. This ratio, or to be more precise the growth rate in output, may significantly differ from 10%. For $r > \rho$, it approaches 0% with higher subjective discount rate and lower intertemporal elasticity of substitution in consumption ($1/\theta$). As a result, Garrison was wrong in saying that in the Fisherian rice model ‘this modelling technique unavoidably conflates growth rates with interest rates’ ([1988], 2011, p. 170).

The greatest opposition of the pure time preference theorists to the Fisherian examples will be separated into two parts. First, it can be argued that even though rice in our example exhibits physical productivity, it may not possess value productivity (Kirzner, [1993], 2011, 109). In the one-good model, value productivity coincides with physical productivity (Garrison, [1988], 2011, p. 170). Value is usually measured in terms of money or in terms of other commodity. In the single-good model this distinction is immaterial, and the most pressing problems in the theory of interest are therefore obscured.

However, assuming that more is always preferred to less, one present unit of rice must have a higher subjective value than one future unit of rice. The reason lies in the fact that the former turns into 1.1 units of the latter. Consequently, if 1.1 units of future rice are valued more than one unit of future rice, present rice must be valued more than future rice - the phenomenon of interest has emerged.

Garrison (1979, p. 146) also claimed that productivity cannot explain the interest phenomenon, because it neglects valuations of acting man. He envisioned a mythical example of an island that is washing away by 20% *per* year. Using the same reasoning as with figs, Garrison implied that the interest rate in terms of the island must be -20%, regardless of valuations of acting man or regardless of the existence of humankind as such.

Let us clarify that Garrison’s approach is not only too radical but also inaccurate. Interest is always a subjective phenomenon that depends on valuations of acting people. There would be no interest in the case of an island that is washing away, because it is not an object of valuing minds. On the other hand, the interest phenomenon will exist in the rice or in the fig economy since rice or figs are valuable goods. They can be exchanged on the intertemporal market. If the valuing mind prefers more rice to less rice, productivity can solely determine the intertemporal price of rice and hence the interest rate in the single-good economy, regardless of the time preference. As a result, even though it is the subjective valuations of acting people that ultimately creates (intertemporal) value of goods, the productivity element might be its crucial determinant, totally neglecting the effect of the preference for present gratification of the given need, *i.e.* the time preference

in sense two. The subjective element is ensured by the intra-temporal valuations of goods - by the diminishing and positive marginal utility, where the positivity of MU implies that more is preferred to less.

The second part of the objection of the PTPT follows Böhm-Bawerk ([1884], 1890) in his criticism of naïve productivity theories. The pure time preference theorists accept that capital is productive in the sense that it produces goods. They also admit that it may produce more goods than is the amount originally invested, as in the example with rice. However, the key problem in the interest theory is why the value of output is eventually higher than the total value of inputs used in the time-consuming process of production. They ask, as Böhm-Bawerk ([1888], 1891) did, why the competition does not eliminate this value difference between inputs (representing future consumption goods) and the eventual output of the resulting consumption goods.

Let us first show how the PTPT theorists would argue in the rice example against the productivity theory. Suppose that 100 present units of rice can produce 110 units of rice next year owing to constant productivity (Kirzner [1993], 2011, p.112). Assume also that people do not obey the fundamental Misesian assumption, so they do not prefer the given satisfaction earlier (ρ is zero). As a result, the emergence of interest, if any, must be only due to physical productivity. Suppose further that both the present price of rice and future price of rice are USD 20. Hence, if 100 units of rice are invested today, the eventual rate of interest will be as follows:³

$$\begin{aligned}\text{interest rate} &= (\text{value of future goods} - \text{value of present goods}) / \text{value of present goods} \\ \text{interest rate} &= (20 \times 110 - 20 \times 100) / (20 \times 100) = 10\%\end{aligned}$$

Böhm-Bawerk ([1888], 1891) tried to explain why the eventual value of output (USD 20 \times 110 units of rice = USD 2,200) is higher than the total value of factors of production invested (USD 20 \times 100 units of rice = USD 2,000). Böhm-Bawerk claimed that the key problem in the interest theory is to explain this value productivity (USD 200), not the physical productivity (additional 10 units of rice). According to the PTPT, this value difference exists due to the time preference and the fact that the production process takes time. In the absence of time preference, competition should gradually eliminate this USD 200 “profit opportunity”.

We assumed at the start of this example that there is no time preference (in the second sense). Hence, Kirzner ([1993], 2011, p. 112) argued that the expected physical increase in output created by the investment of present rice must be eventually imputed to its present price. In the absence of time preference, the present price of rice must increase to USD 22. In equilibrium, the interest (rate) is necessarily 0 = (20 \times 110 – 22 \times 100). There is no value-difference to be explained. As a result, the pure physical productivity cannot induce value productivity, which may emerge only due to time preference. Herbener (2011, p. 44) added that if the time discount rate of people was 5%, the market interest rate must be 5%, not 10%, as would be predicted by the productivity theory.

The key question in this connection is: what is the interest rate Kirzner discussed and what interest rate was presented in Fisher’s examples? It is obvious that Kirzner’s

3 See Cwik (2004, p. 3). However, notice that he made a mistake in his formula - there must be “interest rate” not “interest” on the left hand side of his equation. Furthermore, the formula should be modified as follows: “future value of future goods” and “present value of present goods”.

value-difference argument is about the nominal interest rate. In his attack on the rice example, the nominal interest rate is zero - by investing USD 2,200 now, one can receive USD 2,200 next year. However, what is the real interest rate in this rice economy? If rice is the only good in the economy and the present price level is 1, then the next year price level will fall to $0.91 = (20/22)$. As a result, USD 2,200 earned next year will have 10% ($= 1/0.91 - 1$) higher purchasing power. Next year, one can buy 10% more rice than this year. The real interest rate in this economy is 10%, which is exactly the figure predicted in the previous sections. Even though the nominal interest rate is 0%, the 10% real interest rate is generated by a 10% fall in prices. Although there is no time preference (in sense two), the real interest rate is positive 10%.

It seems that the centre of the entire controversy between the Austrian PTPT and the neoclassical theory is 'only' about the definition of interest. The former school tries to explain the nominal interest rate - the value difference, the premium paid on present money as against future money. For the Austrian authors, the problem is why it is possible to receive USD 2,200 in the future after investing USD 2,000 in the present. Herbener (2011, p. 56) explicitly argued that it is the exchange of present money as against future money that is the core of the theory of interest because it "isolates pure time preference", whereas the exchange ratio between present goods and future goods is not the thing the theory of interest should explain; it is not the rate of interest. The neoclassical school, on the other hand, attempts to explain the real interest rate - the exchange ratio between present goods and future goods. In this theory, the key question is why it is possible to receive 110 units of rice in the future by investing 100 units today.

What approach is the more fundamental? We will argue that in the economic science, real phenomena are of primary importance. Nominal phenomena are just derived from real phenomena after the introduction of money into the given model. In the theory of interest, it is not important how much money one can receive by investing present money in an investment opportunity, in a (roundabout) process of production. The crucial question is how many future real consumption goods one can receive by forgoing present real consumption goods since only real consumption goods are ultimately capable of satisfying human wants (either in the present or in the future), not money.⁴

Let us now support this conclusion with a simple model. The discussion about the nominal interest rate requires introduction of money into the model, which is quite problematic since there is only one single commodity on the island and no intertemporal market. In other words, it is hard to find any role for money in this economy. However, assume that sailors must use money to buy consumption goods both in the present and in the future. Consider some definite amount of money that is proportionately distributed among sailors at time 0. This amount of money is then completely used to buy the total output of consumption goods in the present period. In the second period, the same amount of money is given to the sailors *via* a lump sum transfer to purchase the next period output of consumption goods. Hence, it is assumed that the nominal amount of money is constant over time, and the monetary part of the economy may be described by the following equations: $M = P_0 C_0$ and $M = P_1 C_1$. They represent a simple equation of exchange with

4 Would it be useful to know that the investment of USD 2,000 in rice will earn USD 2,200 or USD 2,000 or USD 2,000,000 next year? Without the knowledge of the evolution of prices in each particular case, this information is almost useless, because it is the problem of allocation of real goods that must be ultimately solved by every agent.

unitary velocity. It can be easily shown that the inflation rate π in this economy is $P_1/P_0 \equiv (1+\pi) = C_0/C_1$ (see Appendix 2).

Using the Euler equation (A1_17) from Appendix 1 and the Fisher definition of the nominal interest rate i , $(1+i) = (1+r) \times (1+\pi)$, the equilibrium rate of interest in this economy is (see Appendix 2):

$$(1+i) = (1+r) \frac{\theta-1}{\theta} (1+\rho)^{1/\theta} \quad (\text{A2}_7)$$

The exogenous real interest rate r is determined by the fixed marginal productivity of capital (rice, hard-tacks, or figs). The exogenous subjective discount rate ρ represents the time preference in sense two, and exogenous parameter θ is the coefficient of the relative risk aversion. It can be shown that $1/\theta$ is equal to the intertemporal elasticity of substitution in consumption.

Equation (A2_7) provides us with key insights about the determinants of the nominal interest rate in this economy. The nominal interest rate is equal to the subjective discount rate ρ only if θ is equal to one (*i.e.* for a logarithmic utility function). Only for a unitary intertemporal elasticity of substitution in consumption, the predictions of this model coincide with the Austrian statement that the interest on money isolates pure time preference.⁵ As a result, the PTPT can be considered as a special case of a more general theory. When $r > \rho$, a lower elasticity of substitution ($\theta > 1$) implies that the equilibrium nominal interest rate is higher than suggested by the PTPT ($i > \rho$), whereas for a higher elasticity of substitution ($\theta < 1$), the nominal interest rate is lower than the subjective discount rate.

For example, in the rice economy with constant marginal productivity ($r = 10\%$) and zero time preference ($\rho = 0$), the investment of USD 2,000 yields USD 2,000 next year only for a logarithmic utility function ($\theta = 1$). Only for this specific combination of parameters, there is no value difference between output and expended inputs, and Kirzner's predictions are confirmed.

A lower elasticity of substitution, *e.g.* $\theta = 2$, leads to a smoother path of optimum consumption (see Euler equation A1_17 in Appendix 1), to a lower growth rate in output (5% according to the Euler equation), and to a lower fall in prices over time ($\pi = -5\%$). In such a case, the investment of USD 2,000 in the present rice with constant real return of 10% results in USD 2,100 earned in the future. As can be seen, interest on money of 5% will emerge in the economy even in the absence of time preference ($\rho = 0$). Note that a positive nominal interest rate is generated in the model with constant marginal productivity of capital and constant money supply only if the real interest rate r is higher than the growth rate in output ($r = 10\% > 5\%$). It can be shown that in the models with infinite horizon, this condition implies a dynamically efficient economy (Phelps, 1961, 1965).

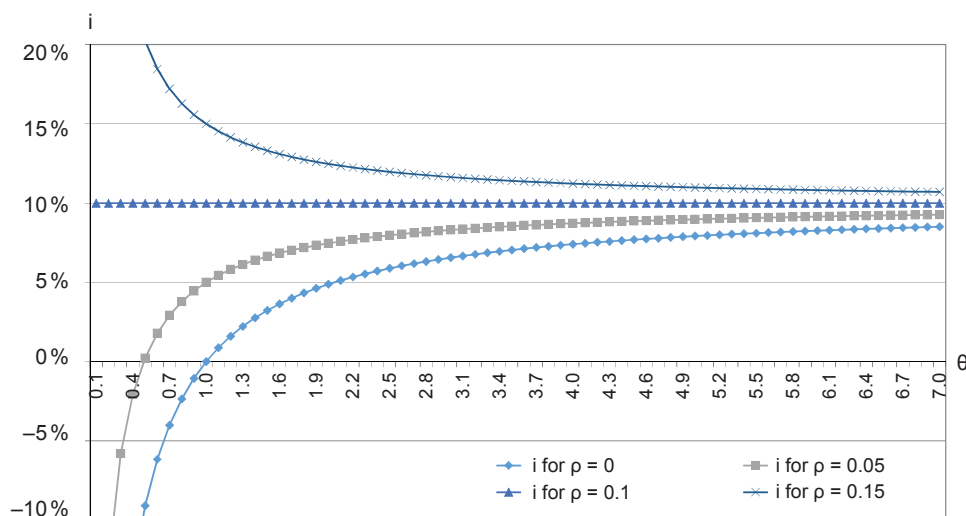
In the hard-tack economy with zero real interest rate, a positive time preference ($\rho > 0$) implies that more hard-tacks are consumed in the present. For a constant money supply, the model predicts a positive inflation and a positive nominal interest. However, nominal interest may be depressed to zero if the preference of sailors to consumption smoothing is very high ($\theta \rightarrow \infty$).

5 There is one more case of equality between ρ and i . If the real rate of interest r is equal to the subjective discount rate ρ (and this possibility is very plausible in an economy with stationary technology and diminishing marginal productivity of capital), parameter θ plays no role. Consumption will be perfectly smoothed, and the inflation rate will be zero. As a result, the nominal rate of interest will then perfectly coincide with the subjective discount rate (and the real rate of interest).

In the economy with deteriorating initial endowment (figs), the equilibrium real rate of interest is negative. This outcome is at odds with the Mises theory that claims that the interest rate cannot decrease below zero as long as the time preference is positive. In the extension of the model that incorporates money, a premium on future goods (*i.e.* a negative real interest rate) is perfectly consistent with positive time preference ($\rho > 0$) and a positive nominal interest rate ($i > 0$), if the elasticity of substitution is high enough (θ is low). This combination implies that a major part of the endowment of figs is consumed in the present, and the resulting high inflation rate raises the interest on money above zero.

Furthermore, it is even possible to set parameters to achieve a negative nominal interest rate in the world with a positive real interest rate and positive time preference. Consider, for example, $r = 10\%$, $\rho = 4\%$, and $\theta = 0.5$. This combination would lead to $i = -1.7\%$, *i.e.* the investment of USD 2,000 today would result only in the total revenue of USD 1,966 next year. However, nobody would be willing to invest money in this case, because its pure hoarding would earn higher nominal return (0% instead of -1.7%). This money hoarding should consequently result in a fall in prices of the factors of production (the price of present rice ready for investment in our example), which would push the nominal interest, *i.e.* the value difference between output and expended inputs, upwards. However, the real interest rate, the growth in output, and the resulting price deflation are ultimately determined by the structural parameters of our model, so the future price of rice must consequently fall by the same percentage as before. Hence, a fall in the present price would not drive up the nominal rate of interest above zero, because the future price would fall accordingly due to the price deflation that is dictated in the model by the values of exogenous parameters. As can be seen, the existence of (constant) money and nominal variables may upset the attainment of the intertemporal equilibrium that would otherwise emerge in a pure barter economy.

Figure 3 | Nominal Rate of Interest i for Various Values of the Subjective Discount Rate ρ and the Coefficient of the Relative Risk Aversion θ



Note: $r = \text{MPK} = 10\%$ by assumption, money supply and velocity are constant

Source: Own simulations

It is usually believed that the money rate of interest cannot fall below zero (neglecting costs to store money) due to the zero lower bound imposed on the nominal interest rate. If we inspect such an economy in more detail, we will see that for the given set of parameters, the growth rate in output of consumption goods (11.9%) exceeds the real interest rate (10%). This is a symptom of dynamic inefficiency in neoclassical growth models. It is well-known that such a state of an economy may lead to curious results (which the negative nominal interest rate certainly is). Figure 3 indicates that this paradox may emerge if the subjective discount rate ρ is low (much lower than 10% if the equilibrium real rate of interest is 10%) and if the intertemporal elasticity of substitution in consumption is high enough (low θ).

The PTPT authors strongly oppose the possibility of a negative (or zero) rate of interest. If our interpretation of this theory is correct, the difference between the value of output and the value of invested capital represents the nominal interest. And this interest cannot (in normal times and under normal conditions) fall below zero.

The foregoing analysis suggested that the PTPT approach might be partly saved from the critique of the neoclassical school if it was reinterpreted as a theory of the nominal interest. However, in such a case, it lost its potential to be accepted as a general equilibrium theory because economists usually seek economic theories elucidating the determination of real phenomena.

4. Conclusion

If the inconsistency indicated in the pure time preference theory is truly present, the productivity element is not only important for the explanation of interest but in case of constant marginal productivity, it can also be the only determinant of the rate of interest. This article tried to show that it is the objective exchange ratio between present goods and future goods, which was associated with the real interest rate, that is the fundamental and genuine centre in the theory of interest. And this magnitude may be completely independent of the size of time preference of acting people.

It seems that the defence of the time-preference theory that arose in the literature put too much emphasis on nominal variables. In other words, pure time-preference theorists considered mainly the interest on money, or the value difference between output and expended inputs. We tried to clarify that such an approach is not accurate, since nominal variables are derived from real variables. As a result, a sound general theory should put aside the veil of nominal variables and focus on the explanation of real phenomena.

Even though it is the subjective valuations of acting man that give value to present goods and future goods and that give the relative value to present goods as against future goods, time preference is neither necessary nor sufficient for such valuations. If more is preferred to less and if the marginal utility is diminishing, the objective element of productivity might be the sole determinant of not only the size of interest but also of the emergence of the interest phenomenon as such. And this interest can be positive, zero, or negative.

We demonstrated that in the two-period model with constant productivity of capital and constant money supply, a positive value difference between output and expended inputs may emerge with no reference to time preference provided that the economy is dynamically efficient. This condition is guaranteed if the discount of future utilities is large enough compared with the constant marginal productivity of capital and/or if the elasticity of substitution is sufficiently low.

Hence, one cannot escape the conclusion that the pure time-preference theory is only a special case of a more general neoclassical framework that incorporates not only the inherent tendency of people to gratify wants as soon as possible, but also the flow of their income over time that may critically depend on the objective element of productivity.

Appendix 1

In Appendix 1, we will solve the optimization problem of a representative shipwrecked sailor from Section 2, who lives for two periods. His budget constraint in the first period is represented by Equation (A1_1):

$$A = C_0 + S_0 \quad (\text{A1}_1)$$

This equation states that the initial stock of hard-tacks A can be used either for present consumption C_0 or some part can be also saved, S_0 . Saved hard-tacks might be lent to somebody else for the interest rate r . Thus, the budget constraint in the next period, which assumes no debts or assets at the end of his life, is as follows:

$$C_1 = S_0(1+r) \quad (\text{A1}_2)$$

The only source for consumption in the next period is the amount of saving from the previous period increased by the accrued interest (if it exists). Inserting the budget constraint of the present period (A1_1) to the budget constraint of the future period (A1_2) will give us his intertemporal budget constraint:

$$C_1 = (A - C_0)(1+r) \quad (\text{A1}_3)$$

Rearranging the terms gives us the key idea that the flow of consumption in the present value may not exceed the initial endowment of hard-tacks:

$$C_0 + \frac{C_1}{1+r} = A \quad (\text{A1}_4)$$

His objective is to find the optimum path of consumption so as to maximize his lifetime utility function (A1_5):

$$U = u(C_0) + \frac{u(C_1)}{1+\rho} \quad (\text{A1}_5)$$

subject to his intertemporal budget constraint (A1_4). Let us set up a simple Lagrange function:

$$L = u(C_0) + \frac{u(C_1)}{1+\rho} + \lambda(A - C_0 - \frac{C_1}{1+r}) \quad (\text{A1}_6)$$

The first order conditions for optimum consumption are:

$$\frac{\partial L}{\partial C_0} = u'(C_0) - \lambda = 0 \quad (\text{A1}_7)$$

$$\frac{\partial L}{\partial C_1} = \frac{u'(C_1)}{1+\rho} - \lambda \frac{1}{1+r} = 0 \quad (\text{A1}_8)$$

$$\frac{\partial L}{\partial \lambda} = A - C_0 - \frac{C_1}{1+r} = 0 \quad (\text{A1}_9)$$

Optimum levels of consumption should be indicated by star (C_0^* and C_1^*). However, we will omit this notation for simplicity, apart from the last equation (A1_17) below. From (A1_7) and (A1_8), it is perfectly clear that the optimum is where the marginal rate of substitution in consumption MRS is equal to the interest factor ($1 + r$), or that the marginal rate of time preference $\varepsilon \equiv MRS - 1$ is equal to the interest rate r :

$$\frac{u'(C_1)}{1 + \rho} = u'(C_0) \frac{1}{1 + r} \quad (\text{A1}_{10})$$

$$\frac{u'(C_0)}{\frac{u'(C_1)}{1 + \rho}} \equiv MRS = 1 + r \quad (\text{A1}_{11})$$

$$\frac{u'(C_0)}{\frac{u'(C_1)}{1 + \rho}} - 1 \equiv MRS - 1 \equiv \varepsilon = r \quad (\text{A1}_{12})$$

From (A1_11), the Euler equation can be easily derived:

$$\frac{u'(C_0)}{u'(C_1)} = \frac{1 + r}{1 + \rho} \quad (\text{A1}_{13})$$

For the CRRA utility function (A1_14):

$$U = \frac{C_0^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_1^{1-\theta}}{1-\theta} \quad (\text{A1}_{14})$$

(A1_11) turns out to be:

$$MRS \equiv \frac{\left(\frac{1}{C_0}\right)^\theta}{\frac{\left(\frac{1}{C_1}\right)^\theta}{(1+\rho)}} = 1 + r \quad (\text{A1}_{15})$$

$$\left(\frac{C_1}{C_0}\right)^\theta (1 + \rho) = 1 + r \quad (\text{A1}_{16})$$

In addition, the corresponding Euler equation, which describes the optimum path of consumption, is as follows:

$$\frac{C_1^*}{C_0^*} = \left(\frac{1 + r}{1 + \rho}\right)^{1/\theta} \quad (\text{A1}_{17})$$

Appendix 2

In Appendix 2, money will be introduced into the model, as was described in Section 3:

$$M = P_0 C_0 \quad (\text{A2}_1)$$

$$M = P_1 C_1 \quad (\text{A2}_2)$$

Equations (A2_1) and (A2_2) represent a simple equation of exchange with unitary velocity. Let us now derive the exact formula for the nominal interest rate in our simple model. From equations (A2_1) and (A2_2), it can be seen that:

$$P_1/P_0 \equiv (1+\pi) = C_0/C_1 \quad (\text{A2}_3)$$

Using the Euler equation (A1_17) from Appendix 1, equation (A2_3) yields:

$$(1+\pi) = \left(\frac{1+\rho}{1+r} \right)^{1/\theta} \quad (\text{A2}_4)$$

If we define the nominal interest rate i in accordance with the Fisher theory:

$$(1+i) = (1+r) \times (1+\pi) \quad (\text{A2}_5)$$

we can conclude that the nominal interest rate in our model is:

$$(1+i) = (1+r) \left(\frac{1+\rho}{1+r} \right)^{1/\theta} \quad (\text{A2}_6)$$

$$(1+i) = (1+r)^{\frac{\theta-1}{\theta}} (1+\rho)^{1/\theta} \quad (\text{A2}_7)$$

References

- Böhm-Bawerk, E. von [1884] (1890). *Capital and Interest*. New York: McMillan and Co.
- Böhm-Bawerk, E. von [1888] (1891). *Positive Theory of Capital*. New York: G. E. Stechert & Co.
- Block, W. (1978). The Negative Interest Rate: Toward a Taxonomic Critique. *Journal of Libertarian Studies*, 2(2), 121–124.
- Brown, H. G. (1913). The Marginal Productivity versus the Impatience Theory of Interest. *The Quarterly Journal of Economics*, 27(4), 630–50.
- Cwik, P. (2004). A Defense of the Traditional Austrian Theory of Interest. Mimeo. [2014-8-22]
Dostupné z: <https://mises.org/journals/scholar/cwik.pdf>
- Fisher, I. (1930). *Theory of Interest*. New York: The Macmillan Company.
- Garrison, R. W. (1979). In Defence of the Misesian Theory of Interest. *Journal of Libertarian Studies*, 3(2), 141–150.
- Garrison, R. W. [1988] (2011). Professor Rothbard and the Theory of Interest, in Herbener, J. M., ed., *The Pure Time-Preference Theory of Interest*. Auburn, AL: Ludwig von Mises Institute, pp. 159–72.
- Ghez, G., Becker, G. S. (1975). *A Theory of the Allocation of Time and Goods over the Life Cycle*. NBER Chapters, National Bureau of Economic Research, pp. 1–45.

- Hayek, F. A. von [1927] (1984). On the Problem of the Theory of Interest, in McCloughry, R. K., ed., *Money, Capital and Fluctuations: Early Essays*. Chicago, IL: Chicago University Press, pp. 55–70.
- Hayek, F. A. von (1941). *The Pure Theory of Capital*, Chicago, IL: Chicago University Press.
- Herbener, J. M. (2011). Introduction, in Herbener, J. M., ed., *The Pure Time-Preference Theory of Interest*. Auburn, AL: Ludwig von Mises Institute, pp. 11–58.
- Herbener, J. M. (2013). Comment on 'A Note on Two Erroneous Ways of Defending the PTPT of Interest'. *The Quarterly Journal of Austrian Economics*, 16(3), pp. 317–330.
- Hülsmann, J. G. (2002). A Theory of Interest. *The Quarterly Journal of Austrian Economics*, 5(4), 77–110. DOI: 10.1007/s12113-002-1007-6.
- Kirzner, I. [1993] (2011). The Pure Time-Preference Theory of Interest: An Attempt at Clarification, in Herbener, J. M., ed., *The Pure Time-Preference Theory of Interest*. Auburn, AL: Ludwig von Mises Institute, pp. 99–126.
- Knight, F. H. (1941). Professor Mises and the Theory of Capital. *Economica*, 8(32), 409–27. DOI: 10.2307/2550107.
- Lewin, P. (1997). Murray Rothbard on Interest and Capital: An Exercise in Theoretical Purity. *Journal of the History of Economic Thought*, 19(1), 141–59. DOI: 10.1017/s1053837200004727.
- Mises, L. von [1949] (1996). *Human Action: A Treatise on Economics*. 4th Ed. San Francisco: Fox & Wilkes.
- Murphy, R. P. (2003). *Unanticipated Intertemporal Change in Theories of Interest*. PhD. Thesis, New York University.
- Olson, M., Bailey, M. J. (1981). Positive Time Preference. *Journal of Political Economy*, 89(1), 1–25. DOI: 10.1086/260947.
- Phelps, E. S. (1961). The Golden Rule of Accumulation: A Fable for Growthmen. *The American Economic Review*, 51(4), 638–643.
- Phelps, E. S. (1965). Second Essay on the Golden Rule of Accumulation. *The American Economic Review*, 55(4), 793–814.
- Rothbard, M. N. [1962] (2004). *Man, Economy, and State*. Auburn, AL: Ludwig von Mises Institute.
- Seager, H. R. (1912). The Impatience Theory of Interest. *The American Economic Review*, 2(4), 834–51.
- Topan, M. V., Păun, C. (2013). A Note on Two Erroneous Ways of Defending the Pure Time Preference Theory of Interest. *The Quarterly Journal of Austrian Economics*, 16(3), 299–316.