

# OPTIMIZATION OF MUNICIPALITIES WITH EXTENDED COMPETENCE SELECTION

Jaroslav Janáček, Bohdan Linda, Iva Ritschelová\*

## Abstract:

Municipalities with extended administration, in which public administration offices are located, were selected within the frame of public administration reform (Act No. 344/1997 Coll., on the Territorial Arrangement of the State and Establishment of Higher Territorial & Administrative Units). These municipalities with extended administration partly substitute the function of district authorities. The selection of municipalities was carried out on the basis of criteria set by the government and political subjects. From the point of view of citizens, the most important criterion for the dislocation of public administration branch offices is the transport availability. Nevertheless, transport availability was not paid relevant attention in the decision making process in question. This fact becomes more and more important in connection with the growing problems related to ensuring regional transport. The paper deals with the creation of a mathematical model of the optimisation of branch offices dislocation and subsequent implementation. The solution of this model has three outputs. The first output is the evaluation of the contemporary state from the point of view of accessibility of branch offices. The second one is the sensibility analysis of accessibility depending on the number of branch offices. The third output is the post-optimisation analysis, which brought about numerical expression of the relationship between the cost of running a branch office and losses due to citizens travelling to municipalities with extended administration.

**Keywords:** public administration reform, municipalities with extended administration, optimal placement of municipalities with extended sphere of authority.

**JEL Classification:** C02, C61, H11, R53

## 1. Introduction

The reform of public administration, which started shortly after the events in November 1989, to some extent still continues also in consequence of the development in the European Union, and it also comprised the change in the territorial & administrative arrangement. Such change was probably the most significant one within the whole reform. The first act relating to the reform of public administration was adopted as

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early as in the year 1990 (Act No. 367/1990 Coll., on Municipalities). However, its proposal was prepared at that time when experience in establishing and managing self-government entities in municipalities did not exist in the Czech Republic (Svoboda *et al.*, 2002). Due to the political situation in the federated republic, the reform receded into the background, and it regained significance as late as after the split of the state in 1992. It was decided to implement a three level system of public administration. The creation of higher territorial & administrative units premised a certain consensus of relevant political parties in Parliament, and that is probably why the territorial & administrative reorganisation took so much time. Particularly the question of the number of future regions was one of the main reasons preventing the adoption of the corresponding act. The Parliament of the Czech Republic adopted the final decision as late as at the end of 1997, and the final number of regions was set at 13 plus Prague - the capital city (Koudelka, 2000). Despite this, political discussions on their number were in progress until the regions started functioning in 2001.

Along with the creation of regions, district authorities were cancelled. Their competences in the sphere of the performance of state administration could be transferred directly and unambiguously neither to individual municipalities nor to regions. Before the district authorities could be cancelled, they had to be replaced by public administration branch offices, where the basic agenda of state administration in their competence was carried out. It is obvious that not all the competences of the district authority could be transferred to each self-governing municipality.

When 76 district authorities had been cancelled (nevertheless the districts still exist as territorial units), then within the framework of the 2<sup>nd</sup> stage of the reform of territorial & public administration, according to Act No. 314/2002 Coll. in force since 1<sup>st</sup> January 2003, approx. 80% of their competences were taken over by 205 municipalities with extended administration<sup>1</sup> (the so-called 3<sup>rd</sup> stage municipalities). On the other hand, the remaining competences of the cancelled district authorities were transferred to the regional authorities (*e.g.* activities of traffic authorities).

The administrative districts of these municipalities with extended administration are outlined by Ministry of Interior Decree No. 388/2002 Coll. with the list of municipalities belonging to it. As a rule, the district of administration of a municipality is formed by part of the former district territory but in some cases the district of administration is identical to the district territory.

Several criteria were set for the selection of municipalities with extended administration:

- Transfer of first instance decision making to the lowest possible level.
- Number of first instance decisions made and issued in individual segments of state administration.
- Effectiveness, quality, and economy of the performance of public administration.
- Requirements for the qualification of office employees due to the competences being transferred.

1 Municipal authorities of the municipalities with extended administrations form a link of transferred competences of state administration between regional authorities and municipal offices. These municipal offices have some additional areas of administrative competences compared with other municipal offices, to be used not only for their own district of administration but usually also for other municipalities in their vicinity.

- The number of inhabitants in the district of administration, approx. 15,000 inhabitants.
- Better availability for citizens.

We can also suppose that despite such clearly set criteria, the selection of municipalities with extended administration was also influenced by the political ambitions of individual members of parliament or their groups. Particularly better availability for citizens, which should have been the main criterion, was probably the last one considered. In fact, it is not known whether any of the location-allocation methods intended to solve such problems, was used. Simply put, location-allocation methods are methods used for establishing service centres (municipalities with extended administration) within the transport network that provide individual nodes of the network (other municipalities) with some services. Such solution also includes the formation of attraction districts of service centres, *i.e.* the formation of disjunctive groups of network nodes characterised by the fact that all the nodes in a given group are served by the same centres (allocation of other municipalities to the municipality with extended administration).

Till the year 2006, some administrative districts of municipalities with extended administration within the framework of a self-governing region extended into more than one district. Nevertheless, according to the Decree No. 513/2006 Coll., on the Change to District Borders, which came into effect as of 1st January 2007, the borders of districts were changed, so that no district of administration of a municipality with extended administration extends outside its own territorial district.

In addition to municipal authorities with extended administration, there also exist authorised municipal authorities (2<sup>nd</sup> stage municipalities) amounting to 389 in total. In some cases the districts of administration, *i.e.* their so-called authorisations, are identical to the district of administration of their extended administration, and in other cases, the administration circuit with extended administration is subdivided into several administrative districts of municipal authorities with extended administration.

## 2. Model Task of the Selection of Municipalities with Extended Administration

As mentioned above, certain criteria were set for the determination of municipalities with extended administration, of which the most important one, from the point of view of the citizen, is the ***criterion of availability***. It means that the citizen assesses the locations of these municipalities according to the comfort with which he/she gets from his/her residence “*j*” to the office “*i*” and back home. In this context, the expression “comfort” is predominantly considered as, *i.e.* the availability of a branch office of public administration.

The system of public administration is being built for the purpose of services provided to the corresponding public, which is a definite but a considerably large set of citizens irregularly distributed in a given territorial area served by a specific transport network. It is clear, in spite of the effort to provide equal access to the services provided by public administration that such equality as far as the availability is concerned, is in principle not possible. On the assumption that the number of service centres is of lower order than that of municipalities, inhabitants of some municipalities will always be at a greater distance from the service centre compared with citizens living in

a municipality where the service centre is established. This inequality of inhabitants can be limited by means of the requirement specifying that the availability of a service centre to any inhabitant must not be worse than a certain limit value. This requirement is derived from a compromise between two points of view, which are: either having the best possible average availability or having the best possible availability for the worst located inhabitant. Another criterion, which is in contradiction with the above-mentioned criteria, is maximum acceptable costs of the creation of the system of public administration. This criterion is often based on the requirement that the number of branch offices cannot exceed certain “*set*” number. It is evident that the more branch offices we establish, the better the average and minimum availability will be, but the costs of their establishment and operation will increase. Creating a good system of public administration means finding an acceptable compromise between the above-mentioned approaches and finding such a distribution within the framework of agreed limits which enables the best possible value of the resulting criterion.

## 2.1 Modelling of Accessibility

Let us look in detail at the concept of time accessibility, and how to understand it. In case the citizen does not possess any means of transportation, he/she is dependent on means of public transport, and he/she is interested in how fast it can take him/her back and forth. This time-based accessibility depends on the existing lines of the collective transport system, the number of transport relations on those lines, and their timetables. Nevertheless, these circumstances are rapidly changing, and therefore their actual state cannot be used as the basis for an optimum selection (from the point of view of time-based availability) of municipalities, to which such extended administration will be transferred. That is why it is necessary to work with a mean time-based accessibility  $TV_{ij}$ , i.e. with the average time necessary for the citizen to get by standard public transport from place  $j$  to place  $i$ . On the other hand, citizens using their own means of transportation are interested in the shortest distance along “*reasonable*” routes (i.e. roads ensuring certain transportation comfort). When we multiply this distance by the average travel speed, we again arrive at the time-based accessibility of individual means of transportation  $TI_{ij}$ .

However, when forming the mathematical model, we need only one time-based accessibility  $T_{ij}$ , which we obtain as a weighted average  $T_{ij} = f_{ij} TV_{ij} + f_{ij} TI_{ij}$ , where  $f_{ij}$  is the proportional part of citizens, who will use public transport, for the journey from point  $j$  to point  $i$ , and citizens  $f_{ij}$  using means of individual transport. It is obvious that an accurate calculation of  $T_{ij}$  is very complicated and for it we should use all the graphic time-tables of all means of public transportation in the country. This task, though complicated, could be carried out.

However, a more serious problem is that, as already mentioned, the conditions and circumstances of the operation of public collective transport change quite quickly, which depends particularly on the economic situation of transportation companies, the Czech Railways, and many other individual transport providers that emerge and disappear in a way, which is difficult to predict.

As the structure of branch offices cannot be changed in dependence on this situation, the criterion of time-based accessibility constructed like this is not suitable

for the distribution and placement of their locations. Nevertheless, with a certain degree of simplification, it holds true that the time distance is proportional to the kilometeric distance; therefore, the criterion of time-based availability can be replaced by the distance-based criterion. The criterial function based on the distance is more suitable for our purpose, as it depends on transport infrastructure, which does not change as quickly as public transport. Advisability of this consideration can be further supported by the fact that the ever increasing vehicle ownership rate moves the focus to the sphere of individual transport. This is in accordance with similar studies in countries with developed motorization, *e.g.* in Germany and others. In addition to it, the criterion modelled like this can be easily transferred to the monetary criterion by mere multiplication by unit costs, which enables to compare direct costs incurred by citizens, and connected to access to the services of the public system, with the costs of its construction, which is also financed by the public, nevertheless indirectly in the form of taxes.

## 2.2 Mathematical Model

Let us indicate the number of all municipalities in the Czech Republic as  $n$ , and the number of municipalities satisfying the requirements for the establishment of a branch office of public administration (in our case, municipalities with extended administration) as  $m$ . The letter  $p$  will indicate a limitation of municipalities with extended administration. The distance of municipalities  $i$  and  $j$  will be denoted as  $d_{ij}$  and the number of inhabitants of municipality  $j$  requiring services in a municipality with extended administration within the framework of current year will be denoted by  $b_j$ . The fact whether municipality  $j$  is to be included in the attraction district of a municipality with extended administration  $i$  (under the attraction district of municipality  $i$  we understand the set of all municipalities  $j$  whose inhabitants will use branch offices for the performance of public administration in municipality  $i$ ) will be expressed as variable  $z_{ij}$ , which only takes the values of 0 or 1. The value of 1 will be taken in case, the municipality  $j$  is included in the attraction district of municipality  $i$ , and the value of 0 otherwise. Analogical binary variable  $y_i$  will identify the fact that the office branch will be established in municipality  $i$ . By means of these decision-making variables, we can model the problem.

First, we will formulate a model, where the optimised criterion expresses the above-mentioned formalised average accessibility. There we define  $c_{ij} = b_j d_{ij}$ , which is the value used in transportation measured in the so called man-kilometres, which in this case expresses requirements for the availability of the branch office located in municipality  $i$  from municipality  $j$ . Then the mathematical model of the task of optimum distribution of a limited number of branch offices of public administration can be written as follows:

$$\text{Minimise} \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} z_{ij} \quad (1)$$

$$\text{Subject to constraints} \quad \sum_{i=1}^m y_i \leq p \quad (2)$$

$$\sum_{i=1}^m z_{ij} = 1, \quad \text{where } j = 1, \dots, n \quad (3)$$

$$z_{ij} \leq y_i, \quad \text{where } i = 1, \dots, m, j = 1, \dots, n \quad (4)$$

$$y_i, z_{ij} \in \{0, 1\}, \quad \text{where } i = 1, \dots, m, j = 1, \dots, n \quad (5)$$

The target function (1) represents the total number of man-kilometres. Constraint (2) means that the number of municipalities with established branch offices does not exceed the set number  $p$ . Constraints (3) ensure that each municipality is allocated to just one branch office. Allocation of a municipality to a place without any branch office is prevented by constraints (4).

The model (1)-(5) can include in several ways the built-in condition of the “equality” of the access of inhabitants to the service, expressed in the requirement stating that for any inhabitant the distance to a service centre should not be longer than the limit value  $D_{max}$ . This can be carried out, e.g. by enlargement of a system of constraints (Janáčková-Szendreyová, 2003).

$$d_{ij} z_{ij} \leq D_{max} y_i, \quad \text{where } i = 1, \dots, m, j = 1, \dots, n. \quad (6)$$

Another manner of modification of coefficients  $c_{ij}$  of the target function for  $i = 1, \dots, m$ ,  $j = 1, \dots, n$  can be performed by means of prohibitive constant  $C$  according to the following prescription:

$$c_{ij} = b_j d_{ij}, \text{ if } d_{ij} \leq D_{max}, \text{ otherwise } c_{ij} = b_j d_{ij} + C.$$

The applied method of modelling also enables to take into consideration also the third economic criterion, which minimizes the total relevant costs formed partly by direct yearly costs incurred by inhabitants, and connected to their visits to branch offices, and partly by relevant yearly costs connected to the operation or costs of the establishment of branch offices in selected municipalities. Nevertheless, it requires knowledge or a qualified estimation of the following costs:  $f_i$  – yearly operating costs connected to the establishment of a branch office in municipality  $i$ , and  $q$  – average expenditures of the citizen spent on travelling a distance of 1km, which could also include the time loss. However, evaluation of the time loss is very difficult and questionable. In this case, we define  $c_{ij} = q b_j d_{ij}$ , if  $d_{ij} \leq D_{max}$ , otherwise  $c_{ij} = q b_j d_{ij} + C$ , and the whole problem obtains the following form:

$$\text{Minimise} \quad \sum_{i=1}^m f_i y_i + \sum_{i=1}^m \sum_{j=1}^n c_{ij} z_{ij} \quad (7)$$

subject to constraints (3), (4) and (5).

The constraint (2) does not occur in the model of this task due to the cost-orientated target function of the model is formulated so that the optimum solution contains such number and location of branch offices, for which the total relevant costs are minimal. The problem (7), (3), (4) and (5) is known under the name: Uncapacitated facility location problem (Janáček, 2006; Janáček, Buzna, 2007; Erlenkotter, 1978). In (Janáček, 2000) the author showed how to transfer the problem (1)–(5) to form of (7), (3), (4) a (5).



Solving this problem of the model with the matrix of distances including all municipalities in the Czech Republic is a demanding task for calculation. Nevertheless, due to its special structure, it can be solved by a branch and bound algorithm based on the Erlenkotter's approach (Erlenkotter, 1978). The corresponding algorithm has been implemented by one of the authors of this article, and is given *e.g.* in (Janáček, 2006) or (Janáček, Buzna, 2007).

### 3. Computational Study

Calculation experiments with the algorithm that solves the problem (2), (3), (4), and (5) were focused on finding the following answers:

- a) Is the existing number of 205 branch offices optimally distributed and placed from the point of view of the average availability of branch offices? If not, what improvement can be expected with the use of exact methods of resolving of the corresponding task?
- b) How does the change in the number of branch offices influence their average availability?
- c) What is the economical optimum number of branch offices, and what average availability corresponds to it?

The answer to the first question was obtained by means of experiments carried out within the grant supported task (Linda, 2003), when several alternative solutions were studied with the existing number of municipalities with extended administration. As the optimality criterion, the author chose the number of man-kilometres calculated from the number of municipality inhabitants and the distance of the municipality from the nearest branch office. On the assumption that the relationship of the number of visits paid to the branch office and the number of inhabitants is constant, it is a value directly proportional to the number of kilometres travelled by the municipality inhabitants to visit the branch office *per* year. **With the existing arrangement of municipalities with extended administration and their attraction districts, the value of the criterial function was calculated at 48,033,391 man-kilometres.** The results of the four most important alternative solutions of the above mentioned tasks are given in Table 1. The currently existing number of branch offices was retained.

In the first option with the existing number of branch offices, we optimised the attraction districts (allocation of municipalities to individual branch offices). The remaining three options included, in addition to optimisation of the attraction district, also the optimisation of the distribution of branch offices in which the number of municipalities applying for a branch office establishment changed. The number of applicant municipalities was set on the basis of the number of inhabitants in such municipalities. In the case of 300 applicant municipalities, the number of inhabitants amounted to 3,800, and in the case of 600 applicant municipalities, it was 1,800 inhabitants. The last column informs about the number of branch offices that had to be transferred to other municipalities within the framework of optimisation.

Table 1

**Answer to the Question: Is the Current Number of 205 Branch Offices Optimally Placed from the Point of View of the Average Availability of Branch Offices?**

Option	Number of branch offices	Number of applicants	Man-kilometres	Saving %	Number of changes
I	205	205	43,551,852	9.3	-
II	205	300	42,326,952	11.9	22
III	205	600	41,430,339	13.7	25
IV	205	All municipalities	41,370,593	13.9	27

Source: Grant project GAČR No. 402/01/1369 and author's calculations.

The last option is rather demonstrative, and shows that an excessive increase in the number of applicants for the establishment of branch offices does not bring about big savings. Branch offices are then also established in municipalities with small number of inhabitants.

The second question was answered by means of sensitivity analysis of the value of the optimum solution of the problem (1) – (5) to the changes of value  $p$  within the interval 155 to 255 with increment 5. The task considered a list of 6,248 municipalities of the Czech Republic excluding the capital city of Prague, and 431 possible locations of branch offices. The matrix of distances between possible locations of branch offices and the considered municipalities covered a maximum of 569 km and a minimum of 37 km. The results with step 10 are given in Table 2. Together with the optimum solution to each task, the following characteristics are assessed:

- $G$  Lagrange multiplier used in the resulting solution for the relaxation of constraint (2). In this case, it is a parameter that is not directly part of the task solution, but it was obtained as an auxiliary value in the iterative calculation. Despite this, it can be used in a subsequent post-optimisation analysis.
- $F^{apl}$  The value of expression (1) of optimum solution of the problem, in which coefficients  $c_{ij}$  were, due to technical reasons, transferred by rounding-off to 500 man-kilometres before calculation.
- $F^{np}$  The value of the expression obtained by inserting an optimum solution of the problem into expression (1), where coefficients  $c_{ij}$  were calculated in man-kilometres, and as such it is given in the tables.
- $P^{exp}$  The number of branch offices located within the framework of the optimum solution. With the use of the given algorithm, this value equals the value of input parameter  $p$ . Only exceptionally can this value differ by several units from the value of the input parameters. In such case, the algorithm provided a sub-optimum solution with value  $F^{apl}$ , which does not differ from the optimum solution by more than  $G^*(p - P^{exp})$ .
- $Av D$  The average of the average availability of branch offices located in individual districts. In this case, the average availability is defined as the average distance in km of the district inhabitant from the branch office, and it is calculated as a total of man-kilometres in the district divided by the number of persons living in the region.



<i>St D</i>	The standard deviation resulting from average availabilities of branch offices located in individual districts.
<i>Min D</i>	The minimum resulting from average availabilities (average distances in km) of branch offices located in individual districts.
<i>Av B</i>	The average number of persons (given in thousands) in the district.
<i>St B</i>	The standard deviation of the number of persons in individual attraction districts expressed in thousands of inhabitants.
<i>Min B</i>	The minimum of the numbers of persons (in thousands) in individual districts with branch offices.
<i>Max B</i>	The maximum of the numbers of persons (in thousands) in individual districts with branch offices.
<i>Av M</i>	The average maximum distance (in km) between a branch office and municipality in individual districts where branch offices are located.
<i>St M</i>	The standard deviation of maximum distances (in km) between a branch office and municipality in individual districts where branch offices are located.
<i>Min M</i>	The minimum of maximum distances (in km) between a branch office and municipality in individual districts, where branch offices are located.
<i>Max M</i>	The maximum of maximum distances (in km) between a branch office and municipality in individual districts, where branch offices are located.

Table 2

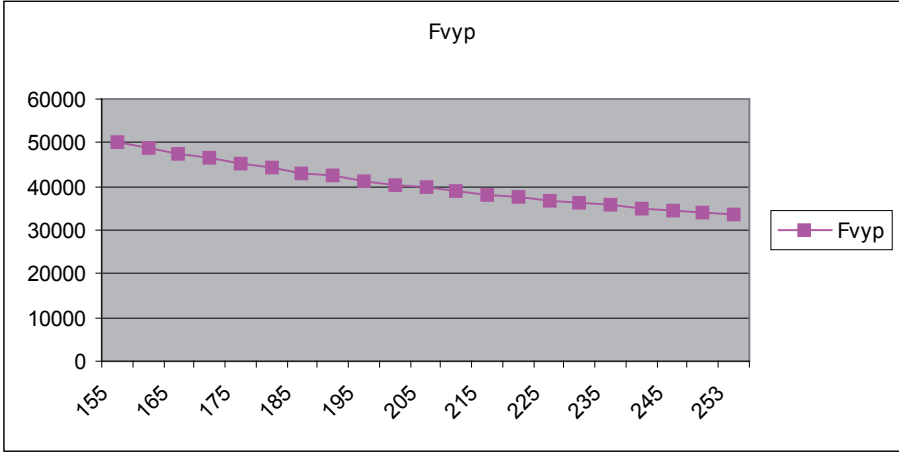
**How Does the Change of the Number of Branch Offices Influence Their Average Availability?**

<i>P</i>	155	165	175	185	195	205	215	225	235	245	255
<i>G</i>	584	522	491	420	381	346	320	295	268	245	221
<i>P<sup>pl</sup></i>	121,448	115,969	110,895	106,303	102,289	98,635	95,309	92,212	89,424	86,843	84,947
<i>P<sup>yp</sup></i>	49,945	47,521	45,300	43,109	41,374	39,658	38,177	36,899	35,604	34,403	33,598
<i>P<sup>exp</sup></i>	155	165	175	185	195	205	215	225	235	245	253
<i>Av D</i>	6.8	6.6	6.3	6.0	5.8	5.6	5.4	5.3	5.1	5.0	4.9
<i>St D</i>	2.6	2.6	2.6	2.5	2.5	2.4	2.3	2.2	2.2	2.2	2.1
<i>Min D</i>	0.6	0.6	0.6	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3
<i>Max D</i>	12.9	12.9	12.9	12.7	12.0	12.0	10.5	10.5	11.0	11.0	11.0
<i>Av B</i>	58.4	54.9	51.8	49.0	46.5	44.2	42.1	40.3	38.5	37.0	35.8
<i>St B</i>	46.9	45.9	44.5	43.5	42.0	41.3	40.6	40.1	39.0	38.1	37.7
<i>Min B</i>	16.9	16.9	14.5	14.0	14.0	13.8	11.43	10.0	10.0	9.2	9.2
<i>Max B</i>	412.3	412.3	412.3	412.3	398.7	398.7	398.7	398.7	396.4	393.2	393.2
<i>Av M</i>	21.4	20.6	19.7	19.2	18.7	18.4	17.9	17.5	17.1	16.9	16.6
<i>St M</i>	5.2	5.0	4.8	5.1	5.1	5.1	5.0	4.9	4.7	4.4	4.4
<i>Min M</i>	9.0	9.0	9.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
<i>Max M</i>	38.0	38.0	38.0	37.0	37.0	37.0	37.0	37.0	37.0	37.0	37.0

Source: author's calculations.

The following Diagram 1 expresses the dependence of the number of man-kilometres (in thousands) on the number of branch offices.

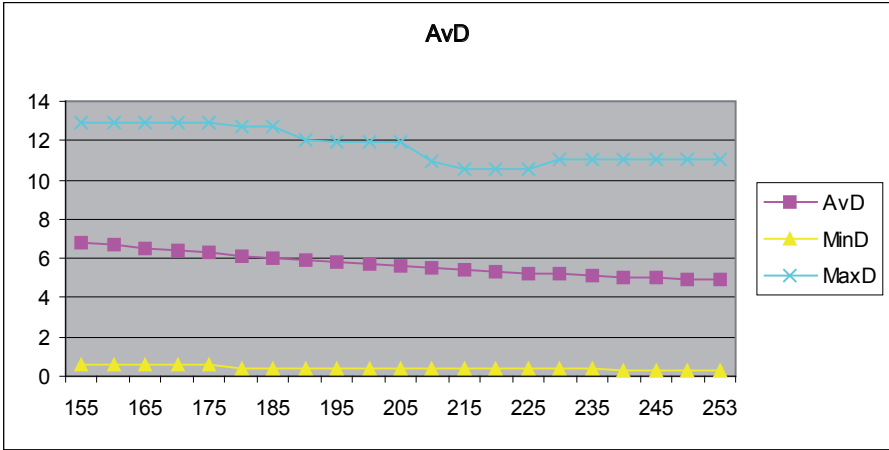
Diagram 1  
Dependence of Availability (in thousands man-kilometres) on the Total Number of Located Offices



Source: author's calculations.

Diagram 2 shows the dependence of the distance of the “average inhabitant” in km from the nearest branch office (AvD). Because this distance in each attraction district of the branch office is different, their average is given, as well as the minimum value (MinD) and maximum value (MaxD) of all attraction districts with branch offices located in the case being solved.

Diagram 2  
Dependence of Average Distances in Individual Districts on the Total Number of Located Offices

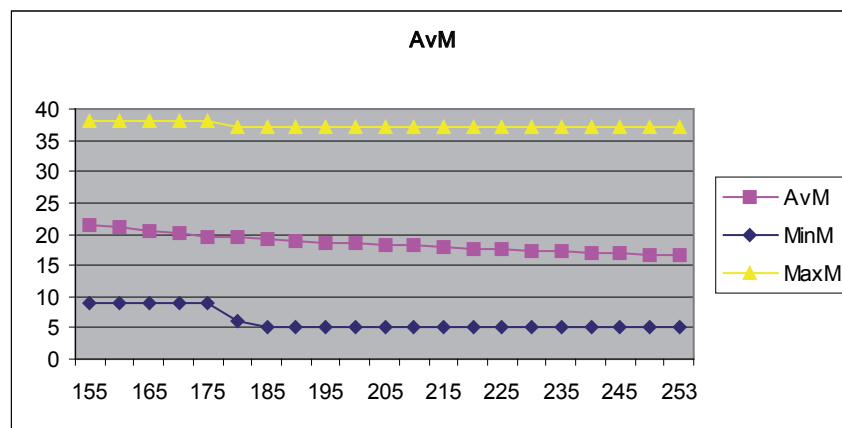


Source: author's calculations

Diagram 3 shows the dependence of the maximum distance (in km) of inhabitants from the nearest branch office (AvM). Because this distance in each attraction district of the branch office is different, their average is given, as well as the minimum value (MinM) and maximum value (MaxD) of all attraction districts with branch offices located in the case being solved.

Diagram 3

Dependence of maximal distances in individual districts on the total number of located offices



Source: author's calculations.

### 3. Post-Optimization Analysis

The presented results of the calculation study do not directly answer the question: "What is the economically optimal number of branch offices?" The reason is that the cost items necessary for quantification of criterion (7) are usually not available. Despite this, use of the post-optimisation analysis and some results obtained in the calculation study can partly lead to the answer to the mentioned question.

If we carry out an analysis of cost coefficient  $c_{ij}$  from criterion (7), we will obtain relationship  $c_{ij} = 2d_{ij}n_j(500r)q$ , where  $d_{ij}$  is the municipality distance  $j$  from the branch office in location  $i$ , and therefore  $2d_{ij}$  is the distance (in km), which the municipality inhabitant travels during a single visit of the branch office. Let  $n_j$  denote the number of municipality inhabitants given in 500s of persons, as was used in the calculation study, frequency  $r$  indicates how many times the average inhabitant visits the branch office, and therefore  $500r$  complies with the annual number of visits related to the  $n_j$  unit. Coefficient  $q$  is the average costs in CZK for one inhabitant travelling one kilometre. Then the relationship between coefficient  $c_{ij}$  of criterion (1), for which the calculation study was carried out, and coefficient  $c_{ij}$  of criterion (7) is  $c_{ij} = c_{ij}(1000rq)$ , where  $c_{ij}$  is given in 500 man-kilometres and  $c_{ij}$  is given in CZK.

When studying the sensitivity of the value of an optimum problem solution (1) – (5) to the change in the  $p$  value in the computational study, as a by-product for each set value  $p$  of the branch offices, we obtained a corresponding value of Lagrange multiplier  $G(p)$  associated with constraint (2). This shadow price measured in 500s of man-kilometres informs about what the change would be in the optimum solution to the task,  $p$  – the right hand side of the constraint (2). And moreover, in the relaxation of constraint (2), multiplier  $G$  will be transferred together with variables  $y_i$  to the target function of the problem being solved, which will acquire the following form after the constant members are removed:

$$\text{Minimise} \quad \sum_{i=1}^m G y_i + \sum_{i=1}^m \sum_{j=1}^n c_{ij} z_{ij} \quad (8)$$

subject to constraints (3), (4) and (5).

If we multiply expression (8), in which optimum solution  $\langle y(p), z(p) \rangle$  and the corresponding value of multiplier  $G(p)$  obtained for the specific  $p$  is substituted, by value  $1000rq$ , we obtain criterion (7), in which the value of  $1000rqG(p)$  corresponds with the yearly costs of one branch office, in which the number of branch offices  $p$  is economically optimal. This enables us to complete a table for a given number of branch offices  $p$ , and for possible values of  $r$  and  $q$ , corresponding to the economically optimal number of branch offices.

As an example, we present Table 3 calculated for  $p = 205$  branch offices, where the annual fixed costs are given in CZK thousands, and Table 4 for annual costs  $f = 358$  (in CZK thousand), where optimum numbers of branch offices are given with accuracy rate = 5. Values  $r$  in Tables 3 and 4 given as numbers of visits of the branch office per inhabitant and year, and values  $q$  given in CZK per one kilometre.

Table 3

**Optimum fixed Costs for Each of the 205 Branch Offices at the Given Frequency of Visits and Unit Travel Expenses.**

$r/q$	125[CZK/km]	2.5[CZK/km]	5[CZK/km]	10[CZK/km]	20[CZK/km]
0.125	54	108	216	433	865
0.25	108	216	433	865	1,730
0.5	216	433	865	1,730	3,460
1	433	865	1,730	3,460	6,920
2	865	1,730	3,460	6,920	13,840
4	1,730	3,460	6,920	13,840	27,680

Source: author's calculations.

Table 4

**Optimum number Of Branch Offices for Fixed Costs Amounting to CZK 358 Thousand at the Given Frequency of Visits and unit Travel Expenses.**

$r/q$	1.3[CZK/km]	1.5[CZK/km]	1.7[CZK/km]	1.9[CZK/km]	2.1[CZK/km]
0.5	155	175	180	195	205
0.6	180	189	200	215	225
0.7	189	205	220	229	245
0.8	205	220	235	245	253
0.9	220	235	245	253	253
1	229	245	253	253	253

Source: author's calculations.

#### 4. Conclusion

As was shown in the computational study, optimisation can be carried out for any number of municipalities with extended administration. It is obvious that an increase in their number results in a decrease in the value of the target function. In a limit case if each municipality has extended administration, the number of criterial functions would be zero, but such state would be unjustifiable for many reasons, particularly from the economic point of view. The above-mentioned procedure enables the set of possible locations of the given number of branch offices to set the location of a given number of branch offices in a way ensuring maximum possible availability for the “average” inhabitant (*i.e.* to ensure a minimum distance of the “average” inhabitant from the nearest branch office). Together with the placement of branch offices, the calculation results will provide optimum attraction districts of individual established branch offices composed of municipalities to be served by the branch office in question. Even the preliminary results given in Table 1 show that the expected improvement exceeds 10% compared to the initial conditions.

Nevertheless, the authors are aware of the fact, that the availability of transport services, though an important criterion, is not the only one, and the establishing or cancelling of branch offices is, in addition to the economic aspect, to a great extent also a political matter. That is why no definite proposals were formulated.

Because, in the negotiations at the corresponding administration and administrative level, it is very useful to know the impact of the number of branch offices on the inhabitants, a numerical analysis of the sensitivity of the optimum value of the target function to the number of located branch offices was carried out (see Table 2 or Chart 2). This analysis showed that within the studied scope in the vicinity of the initial state, the addition of another 10 branch offices would increase the availability for the average inhabitant maximally by 0.2 km, which is naturally connected with the need to cover the operating expenses of these branch offices.

As the idea of suitability or unsuitability of the discussed number of branch offices can be seen best on the basis of deviation from a balanced state between the costs that are directly paid by inhabitants, and the costs paid indirectly by means of taxes, the calculation study was complemented by the post-optimisation analysis, which takes up in a simplified manner the relations between individual types of costs. This method therefore provides supporting tools that could be used, in case of political consent, in the revision of the number of branch offices, and their optimum selection.

Another post-optimisation analysis could be a calculation study answering the question: “What is the environmentally optimum number of branch offices?” It should quantify the influence of individual models on the environment, particularly the quantity of emissions produced by transport.

#### 5. Generalization and Further Prospects

The computational study contained in this paper confirms the finding that usage of exact methods of mathematical programming can bring up to ten percent improvement of the service quality in comparison with an intuitive approach. Taking into account the state of current computational technique, we can claim that all necessary preliminaries

are done for a construction of a decision support tool, which can enable for a common user to obtain very good design of service centre location.

Nevertheless, some questions concerning input data stay open. It can be noticed that cost optimal design of such a service system as the described public administration system needs as the fixed charges for service centre location, as the cost of one man-kilometre. Both these costs are hard to estimate, especially for a long planning period. We have shown one approach to these uncertain data, when the sensitivity analysis was performed to obtain a dependence of the numbers of offices on the unit travel cost. Further possibility of mastering this problem may use the theory of fuzzy sets, which offers numerous tools for data uncertainty processing. This approach has not been explored in this paper, but it will be included in our future research.

## References

- Erlenkotter, D.** (1978): "A Dual-Based Procedure for Uncapacitated Facility Location." *Operations Research*, Vol. 26, No. 6, November-December 1978, pp 992–1009.
- Janáček, J.** (2000): "Transport-Optimal Partitioning of a Region." *Communications – Scientific Letters of the University of Žilina*. 2000, Vol. 2, No. 4, pp 35–42.
- Janáček, J.** (2006): *Optimalizace na dopravních sítích*. 2<sup>nd</sup> Edition. Žilina: EDIS – University Publisher, 2006. 248 pp.
- Janáček, J., Buzna, L.** (2007): *Facility Location in Distribution Systems*. Žilina: EDIS - University Publisher, 2007. 142 pp.
- Janáčková, M., Szendreyová, A.** (2003): "Transportation Management Optimisation with Time restriction." In: 9. Medzinárodná vedecká konferencia. Kvantitatívne metódy v ekonómii. Bratislava, 2003.
- Koudelka, Z.** (2000): *Obce a kraje*. Praha: Linde, 2000.
- Linda, B. et al.** (2001–2003): *Optimalizace rozhodovacích procesů ve veřejné správě s důrazem na regiony*. Grantový projekt GAČR č. 402/01/1369, Pardubice, 2001–2003
- Ritschelová, I. et al.** (2007–2011): *Indikátory pro hodnocení a modelování interakcí mezi životním prostředím, ekonomikou a společností*. Grantový projekt MŽP ČR MoE SP/4i2/60/07, Pardubice, 2007–2011
- Svoboda, K., Grospič, J., Vedral, J., Plíšek, M.** (2002): *Územní samospráva a státní správa*. 2<sup>nd</sup> Edition. Praha: Eurounion, 2002.